

Math 152 - Computer Project 2 - Spring 2005 - Revised

Optimization and Foraging Theory

This project requires you to use the ideas of Section 14 regarding maximization of a function, based upon the assumption that evolution has acted to generate highly efficient foragers. Highly efficient here is defined to mean that animals searching for food which are more able to rapidly obtain a high food intake rate (food eaten per unit time), will be more likely to survive and reproduce. Thus, if the characteristics that lead to high food intake efficiency (which may depend upon speed, visual or hearing skills, size, etc.) are heritable, then the characteristics which lead to higher efficiency will become more prevalent (increase in frequency) in the population. This area of science is called optimal foraging theory and for a readable description, read David Stephens and John Krebs' book "Foraging Theory".

One of the main areas of foraging theory deals with animals which move from "patch" to "patch" across a landscape, depleting the food in each patch the longer they stay there. Patches could be individual plants with nectar in the plant's flowers that is eaten by bees, or individual plants with seeds that are eaten by birds. One question in foraging theory is to deduce the rules for when an individual will leave a patch to search for a new one. Numerous complications arise in this, but we will consider only a very simple situation, in which (1) there are no interactions with other foragers, so depletion of food in a patch is due only to the single forager under consideration; (2) there is no randomness in the environment, so that each patch is identical in terms of how much resource it has before foraging starts; (3) once the patch is depleted it stays depleted; (5) there are many patches available and the travel-time of the forager between patches is constant; (6) the food available in the patch decreases exponentially as the forager spends time there.

Let

T = time spent in a patch before leaving

M = time it takes to move between two patches

c = decay rate of available food in patch once foraging starts

K = amount of food in patch before any food has been eaten there

Then the total amount of food eaten in a patch is

$$K - K e^{-c T} = K (1 - e^{-c T}) \quad (1)$$

and the total time over one foraging "bout" is $T + M$. Thus the food intake rate is

$$\frac{K (1 - e^{-c T})}{T + M} \quad (2)$$

Use Maple to do the following:

(a) Find an equation for the "optimal" time, T^* , at which a forager should leave a patch in order to maximize the ratio in (2).

(b) Let q = your birthday as fraction of time through the year (so that if your birthday is January 10, $q=10/365$ and if your birthday is December 30, $q=364/365$). Letting $K=1$ and $c=10*q$, plot the optimal time T^* as a function of M for M varying from 1 to 20.

(c) Again letting $K=1$, do a 3-dimensional graph of T^* as a function of M (varying from 1 to 20) and c (varying from 0 to 10).

(d) The theory of optimal foraging gives what is called the "marginal value theorem" which argues that the optimal time to leave a patch (T^* in our case) is chosen so that the marginal gain in a patch at that time (this is the derivative of (1) at time T^*) is equal to the long-term average rate of gain (this is (2) calculated at T^*). Using the values from (b), determine if the marginal value theorem is correct for your case. Do this by plotting the value of (2) at T^* as a function of M for M varying from 1 to 20 and comparing this to the plot of the derivative of (1) with respect to T evaluated at T^* as a function of M for M varying from 1 to 20.

NOTE: You will need to use a command such as `fsolve` in Maple to do the above since you will need to solve a transcendental equation. A sample Maple worksheet that shows you how to do all of the above for a different function (1) and (2) is on the course home page. Use this to assist you in doing the project.