Math 151 – Sample Exam II Answers– Fall 2007 – Louis Gross

1. (a) $\begin{bmatrix} -3 & 11 \\ -8 & -10 \end{bmatrix}$ (b) Not defined (c) $\begin{bmatrix} -5 & 1 \\ 4 & 3 \\ -7 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 15 & 12 & 5 \\ 33 & 18 & 16 \\ -1 & -10 & -15 \end{bmatrix}$

2. Eigenvalue -1 has eigenvector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and eigenvalue 3 has eigenvector $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

3. (a) $\begin{bmatrix} F \\ S \end{bmatrix}_1 = \begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix} \begin{bmatrix} F \\ S \end{bmatrix}_0$ (b) $\begin{bmatrix} F \\ S \end{bmatrix}_2 = \begin{bmatrix} 90 \\ 18 \end{bmatrix}$

   (c) 3 is the dominant eigenvalue and so it is the long-term growth rate
and at time 101 there would be approximately 3000 individuals present

   (d) the long-term fraction in each class would be he eigenvector for eigenvalue 3
which is $\begin{bmatrix} 30/31 \\ 1/31 \end{bmatrix}$ so the long-term fraction which are F is 30/31 and the S is 1/31

4. (a) $A(t) = 2\,(5^{t/3})$

   (b) A(0)=2   doubling time is $\dfrac{3\log(2)}{\log(5)} = 1.3\,days$

   (c) $\dfrac{3\log(250)}{\log(5)} = 10.3\,days$

5. (a) $P = \begin{bmatrix} .75 & 0 & .01 \\ .2 & .97 & 0 \\ .05 & .03 & .99 \end{bmatrix}$ (b) $\begin{bmatrix} 750 \\ 209.7 \\ 50.3 \end{bmatrix}$ (c) the long term fraction in each state would

be the eigenvector corresponding t the eigenvalue 1 – it is $\begin{bmatrix} .03 \\ .2 \\ .77 \end{bmatrix}$