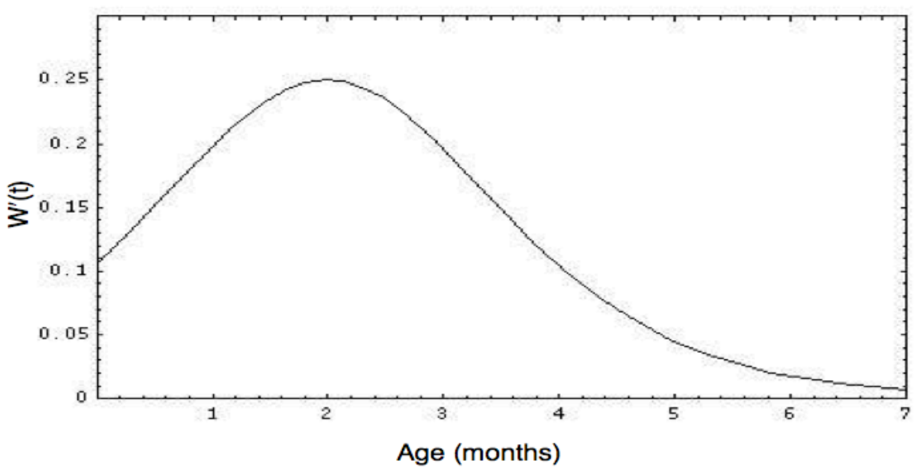
Math 152 – Sample Final Exam Brief Answers – Spring 2016 – Louis Gross

1. (a) By definition $W'(a) = \lim_{h\to 0} \frac{W(a+h)-W(a)}{h}$ gives the instantaneous growth rate of the weight of a fish of age a. The units are kg/month

(b) $W'(2)$ is the slope of the tangent line to the graph of W(a) at a=2 and this slope is approximately .25 kg/month.

(c)



2. (a) At birth, using the equation for the derivative, $L'(0) = .2(40-3) = 7.4$ cm/month

(b) Since $\lim_{t\to\infty} L(t) = L_\infty = 40$ we are looking for the age at which the fish has length 20cm. So we want to find a such that L(a)=20. But first we need to use the fact that L(0)=3 to find t_0. So $L(0) = 3 = 40(1-e^{-.2(0-t_0)}) = 40(1-e^{.2t_0})$ so $t_0 = 5\ln(\frac{37}{40}) = -.39$
Then $L(a) = 20 = 40(1-e^{-.2(a+.39)})$ so $a = -5\ln(.5) - .39 = 3.08$ months.

3. Separate variables and integrate to get $y(t) = 4e^{t^2+t}$

4. (a) We want to find α so that $\int_0^1 (\alpha + x)\,dx = 1$ which gives $\alpha = \frac{1}{2}$

(b) $P[X \le \frac{1}{2}] = \int_0^{1/2} (\frac{1}{2} + x)\,dx = \frac{3}{8}$

(c) $E[X] = \int_0^1 x(\frac{1}{2} + x)\,dx = \frac{7}{12}$

(d) The median has value m if $\frac{1}{2} = \int_0^m (\frac{1}{2} + x)\,dx = \frac{m}{2} + \frac{m^2}{2}$ so there are two possible m values that satisfy this quadratic equation but only one of them is feasible so

$m = \frac{\sqrt{5}}{2} - \frac{1}{2} = .618$

5. (a) $\frac{dx}{dt} = .02\frac{kg}{l} \cdot 100\frac{l}{hr} - \frac{x}{20000}\frac{kg}{l} \cdot 100\frac{l}{hr} = 2 - \frac{x}{200}$

(b) $x(0) = .005\frac{kg}{l} \cdot 20000l = 100kg$

(c) We can separate variables to get $\int \frac{1}{400 - x}dx = \int \frac{1}{200}dt$ and then integrating we get $x(t) = 400 - Ke^{-\frac{t}{200}}$ and using x(0)=100 gives K=300 so $x(t) = 400 - 300e^{-\frac{t}{200}}$

(d) We want to find a time T so that x(T)=200 so $200 = 400 - 300e^{-\frac{T}{200}}$ so $T = 200\ln(1.5) = 81.09hr$

6. If we set $\frac{dN}{dt} = 0$ we see that this means $N(1 - N^2) = 0$ so N=0, or N=1 or N=-1 but since N is a density of cells it cannot be negative so the only feasible equilibria are N=0 and N=1. Note that if N is slightly larger than 0, $N'(t) > 0$ for example if N=.1 then $N'(t) = \frac{2(.1)}{1.01} - .1 = .098$ so N=0 is not a stable equilibrium because if the population were close to 0 but positive, the population would grow. However if the population were slightly below 1, then $N'(t) > 0$ and if the population were slightly greater than 1, then $N'(t) < 0$ so N=1 is a stable equilibrium. You can check this by noting that if N=.99 then $N' = \frac{2(.99)}{1 + .99} - .99 = .005$ and if N=1.01 then $N' = \frac{2(1.01)}{1 + 1.01} - 1.01 = -.005$

7. $\int_0^2 (4x - 2x^2)\,dx = \frac{8}{3}$

8. (a) density is maximum when $f'(x) = 0$ which means $x = \frac{1}{\sqrt{6}} = .408m$

(b) $\int_0^1 6xe^{-3x^2}\,dx = 1 - e^{-3} = 1 - .05 = .95$

9. If A(t) = area of the fungal culture at time t, then $A'(t) = kA(t)$ implies $A(t) = A(0)e^{-kt}$ So measure A(t) at several times (e.g. at times t_1, t_2 , t_3 , etc.) and since $\ln(A(t)) = \ln(A(0)) - kt$ then plot t_i versus $A(t_i)$ on semilog scale. If this gives a linear graph, accept the hypothesis, otherwise reject it. (Note: you could calculate R^2 from the linear regression and reject the hypothesis if R is not above .5 say)

10. (a) Use integration by parts to get $-\frac{3}{4}e^{-4x}(x+\frac{1}{4})+C$

(b) $\frac{1}{2}+\frac{1}{\pi}$

11. (a) $y' = 4\ln(2t+1)+\frac{8t}{2t+1}$

(b) $g'(x) = \frac{1}{(x+1)^2}$

12. (a) $K = \lim_{a\to\infty} B(a) = 150$ tons/hectare

(b) $B'(a) = \frac{13500e^{-\frac{a}{10}}}{(10+90e^{-\frac{a}{10}})^2}$

(c) $B(a) = 75$ when $a = 10\ln 9 = 21.97$ years

(d) $B'(a)$ will be maximized when $B''(a) = 0$ so

$B''(a) = rB'(\frac{K-B}{K}) + rB(-\frac{B'}{K}) = rB'(1-\frac{2B}{K})$ so this = 0 when $B = \frac{K}{2}$