Math 152 - Sample Exam 1 - Spring 2004

Note: This sample exam is longer than the actual exam will be - it is designed to give you some additional proactic problems.

1. Find the following limits, if they exist. If they don't exist, state so.

(a) $\lim_{y \to \infty} \frac{4 - y}{2y + 3}$ (b) $\lim_{x \to 3} \frac{4x + 2}{x + 1}$

(c) $\lim_{h \to 0} \frac{h^2 - 2h}{3h^2 + h}$ (d) $\lim_{x \to -\infty} \frac{5x^2}{2x^2 + 3x - 1}$

(e) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ (f) $\lim_{y \to \infty} \frac{3y}{2y + \sqrt{y}}$

(g) $\lim_{x \to -1} \left(8x^2 + \frac{x - 1}{x + 1} \right)$

2. Sketch the graphs of three different functions which are not continuous at $x = 2$, being sure to give the equations for the functions you are graphing.

3. For each of the following functions, state where the function is continuous:

(a) $f(x) = \begin{cases} x + 1 & \text{for } x \le -1 \\ x^2 & \text{for } -1 < x \le 1 \\ x & \text{for } x > 1 \end{cases}$ (b) $g(x) = \begin{cases} x^2 & \text{for } x \le 0 \\ 1/x^2 & \text{for } 0 < x < 1/2 \\ x-1 & \text{for } x \ge 1/2 \end{cases}$

4. Find the derivative of each of the following functions:

(a) $f(x) = 3x^2 - x + 4/x$ (b) $g(y) = \cos(1 - 3y)$

(c) $y(t) = (t^2 + 4t)^5$ (d) $f(z) = 3z \exp(z^2 - 4z)$

(e) $g(x) = \ln(x^2 + 3x + 1)$ (f) $f(t) = (3t - 1) / (t^2 + 1)$

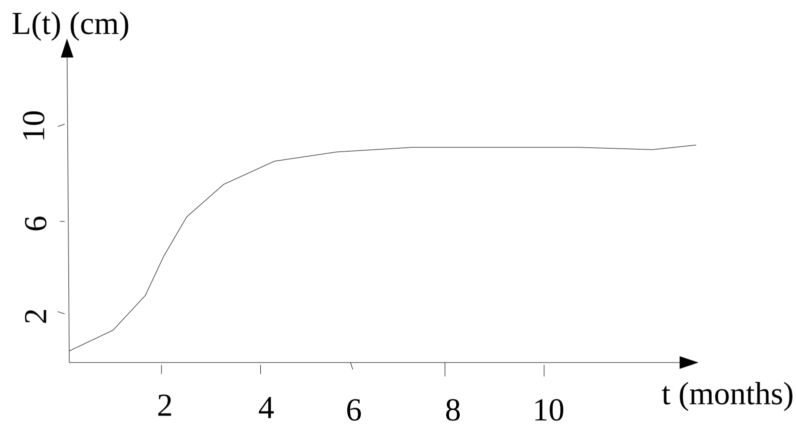
5. Find the equation of the line which is tangent to the graph of $y = 3x^2 + 2x - 3$ at $x=1$.

6. Suppose L(t) gives the length of a fish in cm at time t, where t is measured in months since hatching.

(a) Give the definition of the derivative of L(t) at time 2 months, $L'(2)$.

(b) Explain in words what $L'(2)$ means, and give its units.

(c) If L(t) looks like the below graph, sketch a graph which indicates how $L'(t)$ changes from time 0 to time 10 months.



7. A particle's position at time t is given by $f(t) = (t^2 + 2t)^{1/2}$ where t is measured in seconds and f(t) is measured in meters. Give a function for the instantaneous speed of the particle and one for its acceleration. What is the particle's speed at time 4 seconds? Is the particle accelerating or deccelerating at time 4 seconds? What is the velocity of the particle after a long time?

8. A reptile's core body temperature in ºC is found to vary through a day according to $T(t) = 20 + 10 \cos(\pi t / 12)$ where t is in hours and t=0 corresponds to noon.

(a) Is the core temperature increasing or decreasing at 4PM?

(b) What is the rate of change of body temperature at 6 PM?

(c) At what times of day is the rate of change of core body temperature equal to zero?

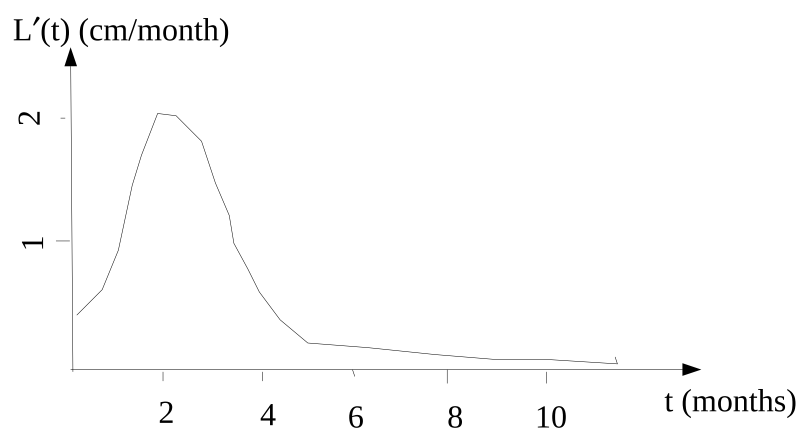
Math 152 - Sample Exam #2 - Spring 2004 (Answers)

1. (a) -1/2 (b) 7/2 (c) 1/3 (d) 5/2 (e) 12 (f) 3/2 (g) Doesn't exist
2. Many possible choices, including $f(x) = 1/(x-2)$, $g(x)=\ln(x-2)$,
   $y(x) = x^2$ for $x<2$ and x for $x>2$
3. (a) Continuous on $(-\infty, -1) \cup (-1, \infty)$ (b) Continuous on $(-\infty, 0) \cup (0,.5) \cup (.5,\infty)$
4. (a) $f'(x) = 6x - 1 - 4/x^2$ (b) $g'(y) = 3 \sin(1 - 3y)$ (c) $y'(t) = 10(t + 2)(t^2 + 4t)^4$
   (d) $f'(z) = 3(2z^2 - 4z + 1)\exp(z^2 - 4z)$ (e) $g'(x) = (2x + 3) / (x^2 + 3x + 1)$
   (f) $f'(t) = (3 + 2t - 3t^2) / (t^2 + 1)^2$
5. slope = 8, point is (1,2), tangent line is $y = 8x - 6$
6. (a)

$$L'(t) = \lim_{h \to 0} \frac{L(h+2) - L(2)}{h} = \lim_{t \to 2} \frac{L(t) - L(2)}{t - 2}$$

(b) $L'(2)$ is the instantaneous rate of growth in length of a fish exactly at age 2 months. Its units are cm/month.

(c)



7. $v(t) = (t + 1)/(t^2 + 2t)^{1/2}$, $a(t) = -(t^2 + 2t)^{-3/2}$, $v(4) = 5/\sqrt{24}$ m/s,
   deccelerating at time $t = 4$ since $a(4)<0$, $\lim_{t\to\infty} v(t) = 1$ m/s
8. (a) $T'(t) = -(5/6)\pi \sin(\pi t/12)$ so $T'(4) = -(5/6)\pi \sin(\pi/3) = -(5\sqrt{3})\pi/12 < 0$ so temperature is decreasing at 4PM
   (b) $T'(6) = -(5/6)\pi \sin(\pi/2) = -(5/6)\pi$ ºC/hr
   (c) $T'(t) = 0$ when $t = 0$ and $t = 12$ so temperature not changing at noon and midnight