Math 151 – Sample Exam II Answers– Fall 2015 – Louis Gross

1. (a) $\begin{bmatrix} -3 & 11 \\ -8 & -10 \end{bmatrix}$ (b) Not defined (c) $\begin{bmatrix} 4 & 3 \\ 38 & 11 \end{bmatrix}$ (d) $\begin{bmatrix} 15 & 12 & 5 \\ 33 & 18 & 16 \\ -1 & -10 & -15 \end{bmatrix}$

2. Eigenvalue -1 has eigenvector $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and eigenvalue 3 has eigenvector $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$

3. (a) $\begin{bmatrix} F \\ S \end{bmatrix}_1 = \begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix} \begin{bmatrix} F \\ S \end{bmatrix}_0$ (b) $\begin{bmatrix} F \\ S \end{bmatrix}_2 = \begin{bmatrix} 90 \\ 18 \end{bmatrix}$

(c) 3 is the dominant eigenvalue and so it is the long-term growth rate and at time 101 there would be approximately 3000 individuals present

(d) the long-term fraction in each class would be he eigenvector for eigenvalue 3 which is $\begin{bmatrix} 30/31 \\ 1/31 \end{bmatrix}$ so the long-term fraction which are F is 30/31 and for S it is 1/31

4. (a) $A(t) = 2\,(5^{t/3})$

(b) A(0)=2 doubling time is $\frac{3\log(2)}{\log(5)} = 1.3\,days$

(c) $\frac{3\log(250)}{\log(5)} = 10.3\,days$

5. The first line calculates the population size in each class after one time step if the population starts with 10 individuals in the first age class and no individuals in the second and third age classes.
The second line calculates the population size in each class after 50 time steps if the population starts with 10 individuals in the first age class and no individuals in the second and third age classes.
The third line calculates the 3 eigenvalues of the matrix P and places them in the diagonal elements of the 3x3 matrix e and calculates the associated eigenvectors for these 3 eigenvalues and places them in the columns of the 3x3 matrix v.

6. (a) $P = \begin{bmatrix} .75 & 0 & .01 \\ .2 & .97 & 0 \\ .05 & .03 & .99 \end{bmatrix}$ (b) $\begin{bmatrix} 750 \\ 209.7 \\ 50.3 \end{bmatrix}$ (c) the long term fraction in each state would be the eigenvector corresponding t the eigenvalue 1 – it is $\begin{bmatrix} .03 \\ .2 \\ .77 \end{bmatrix}$