Math 152 - Sample Final Exam - Spring 2005

7. Find the following:

(a) $\int (3\,x\,e^{-4\,x})\,dx$ (b) $\int_{-1}^{1} \frac{1}{(2-x)^3}\,dx$ (c) $\int \frac{\ln x}{x}\,dx$

8. The length of a fish of age t grows according to
$L'(t) = K\,(L_\infty - L)$ so $L(t) = L_\infty(1 - e^{-K(t - t_0)})$
with $L_\infty = 40$ cm, K = .2/month and L(0)=3 cm. (a) How fast is the fish growing in length at birth? (b) At what age will the fish reach 1/2 of its largest length?

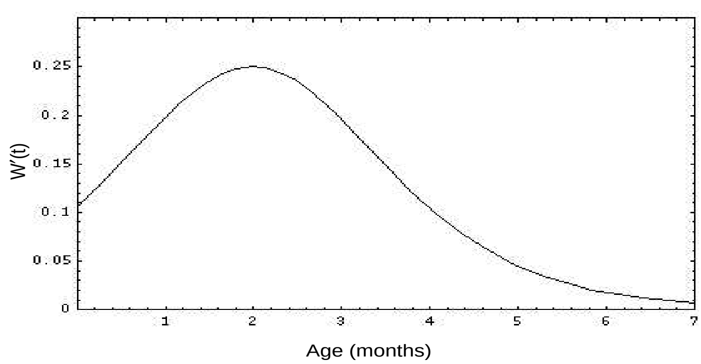
9. How much work is necessary to pump the contents of a tank of liquid of density 40 kg per cubic meter just over the top of the tank if the tank is cone shaped with a circular base of radius 6 meters and a circular top of radius 3 meters with the top being 12 meters above the ground. The tank is half full of liquid at the start (e.g. the level of liquid in the tank is 6 meters above the bottom of the tank). .

10. Find (a) the solution of $y'=(2t+1)\,y$ if y(0)=4.
(b) all solutions of $\frac{dN}{dt} = \frac{N}{2\,t}$

Answers

1.If A(t) = area of fungal culture at time t, A'(t)=kA(t) implies $A(t)=A(0)\,e^{k\,t}$.
So measure A(t) at several times (e.g. $t_1, t_2, t_3, \ldots$) and since lnA(t) = kt+lnA(0), plot t_i versus $A(t_i)$ on semilog paper (equivalent to plotting t_i versus $\ln A(t_i)$) on regular graph paper). If this gives a linear graph, accept the hypothesis, otherwise reject it. (Note: you could calculate R^2 from the linear regression and reject the hypothesis if R^2 is not above .5 say)

2. By Definition $W'(t) = \lim_{h \to 0} \frac{W(t+h) - W(t)}{h}$ gives the growth rate of the fish at a particular instant t, in kg/month and W'(2) is approximately .25 kg/month



3. (a) $y'(t) = 4\ln(2t+1) + 8t/(2t+1)$ (b) $g' = \frac{1}{(x\;\;+1)^2}$ (c) $f' = \frac{-6\,(3y+1)}{(3\,y^2 + 2y)^4}$

(d) $y'(t) = -4\,(\sin 4\,t)\,e^{\cos 4t}$