

## Math 152 - Sample Exam 2 - Spring 2005

**Note that this Exam will cover sections 12-18 in the text but may include a problem similar to those on Exam 1 as well. Thus you should look over Exam 1 and make certain you understand how to do the problems on that exam.**

1. Sketch the graphs of the following functions. State where all relative maxima and minima occur and give the location of any inflection points.

(a) 
$$y = \frac{x^4}{4} - \frac{3x^2}{2} + 1$$

(b) 
$$y = \frac{x^2 - 1}{x^2 + 1}$$

(c) 
$$f(x) = x + 2 - \frac{1}{x - 2}$$

2. The size of web a particular spider makes affects the daily food intake in some sites, with larger webs allowing more food to be captured. However, larger webs require more energy for the spider to build and maintain. For a circular web, assume that the amount of food captured per day is proportional to the radius of the web, but the maintenance and building cost is proportional to the web area. Suppose the daily food intake is 120 calories when the web has radius 10 cm and the cost is 30 calories per day when the web area is  $100 \text{ cm}^2$ . Find the web size that is optimal, which means that it maximizes the daily food intake minus the cost.

3. Solve the following differential equations subject to the given boundary conditions.

(a) 
$$y'(x) = x + 5 \quad y(2) = 8$$

(b) 
$$y''(x) = 2x \quad y'(0) = 0, \quad y(0) = 4$$

4. Find the most general antiderivatives of the following functions.

(a) 
$$f(z) = z^5 - 2z^3 - 1$$

(b) 
$$g(t) = \frac{1}{t^3} + 2\sqrt{t} - 3t^2$$

(c) 
$$h(y) = e^{2y} + \frac{1}{y^3}$$

(d) 
$$y(x) = 2 \sin(3x) - \cos x$$

5. Find the largest and smallest values of each of the following functions on the given intervals and state the x-value at which these occur.

(a)  $f(x) = x + \frac{4}{x}$ , on  $[1, 3]$

(b)  $f(x) = \frac{x}{x^2 + 1}$ , on  $[-2, 2]$

6. The change in a lizard's body temperature when the animal moves from sun to shade (this behavior is called shuttling) is found to be reasonably well described by Newton's Law of Cooling. Thus the rate of change of body temperature is proportional to the difference between body temperature and the temperature of the surroundings. Suppose a lizard's temperature is  $30^\circ C$  when it is in the sun and drops to  $20^\circ C$  after 10 minutes in the shade where the surrounding temperature is  $15^\circ C$ . If  $T(t)$  gives the lizard's temperature at time  $t$  minutes after going from sun to shade, then

(a) Give an equation for  $T'(t)$  as well as the boundary condition.

(b) Find an equation for  $T(t)$  and use it to find  $T(20)$ .

7. For fish such as salmon, the Ricker Model provides an estimate of the number of new "recruits",  $R$  which means the number of fish which will survive to become adults in the next generation, as a function of the number of fish,  $S$ , spawning in the previous generation. The model states that

$$R = aS e^{-bS}$$

For this model, find an equation for the level of spawners,  $S^*$ , which would maximize the number of recruits.

8. State where each of the following functions is concave up, where it is concave down and where it has inflection points.

(a)  $f(x) = x^2 e^{-x}$

(b)  $g(x) = 6x^2 + 3 \ln x$  for  $x > 0$