Math 151- Final Exam - Fall 2006   Lou Gross

Do all your work on the sheets provided, not this question sheet (please use only one side of each sheet). Be sure to SHOW YOUR WORK, and please put your name on each sheet. Please circle the final answer to each problem. Write your TA's name (Erin Bodine, Lauren Wagener or Rachael Miller) on the first page as well as your section number. Point values are in parentheses with a total of 200 points.

1. A 70 kg patient is being treated with an antibacterial drug following surgery to reduce the risk of dangerous infection. The drug is metabolized and excreted in the body so that it decays exponentially with a half-life of 10 hours. The effective dosage range of this drug is from 10 mg per kg of body weight to 15 mg per kg of body weight. The physician has specified that a dose is to be given every 5 hours.
   (a) What fraction of drug in the patient's body will decay between each dose? (8 pts)
   (b) What periodic dose of the drug must be given so the amount in the patient just before each dose is at the lower end of the effective range? (8pts)
   (c) For the situation in (b), what is the loading (Bolus) dose to be given to this patient initially, and what is the amount of the drug in the patient just before each periodic dose? (8 pts)
   (d) For the situation in (b) and (c), sketch a graph of the amount of drug in the patient over the first 20 hours the patient is on the drug. (8 pts)
2. A study of phosphorus levels in a small lake following the development of cottages on the edge of the lake finds that 40% of the phosphorus in the lake is lost each year due to settlement into the sediments and due to uptake by aquatic plants. Initially, before the cottages were built, there was 60 kg of phosphorus in the lake. Each year the effluent from the cottages adds 50 kg of phosphorus to the lake. Let x_n = the amount of phosphorus in the lake n years after the cottages were built.
   (a) Explain why an appropriate equation to describe this system is $x_{n+1} = .6\, x_n + 50$ (5 pts)
   (b) Solve the equation in (a) to give a formula for x_n in terms of n which takes into account the initial condition that $x_0 = 60$. (10 pts)
   (c) What will the amount of phosphorus be in the lake 3 years after the cottages were built and what will be the long-term equilibrium amount of phosphorus in the lake? (10 pts)
3. Solve the following difference equation with the given initial conditions (18 pts)

   $$x_{n+1} - x_n - 6\, x_{n-1} = 12 \qquad x_0 = 10\,, \quad x_1 = -6$$

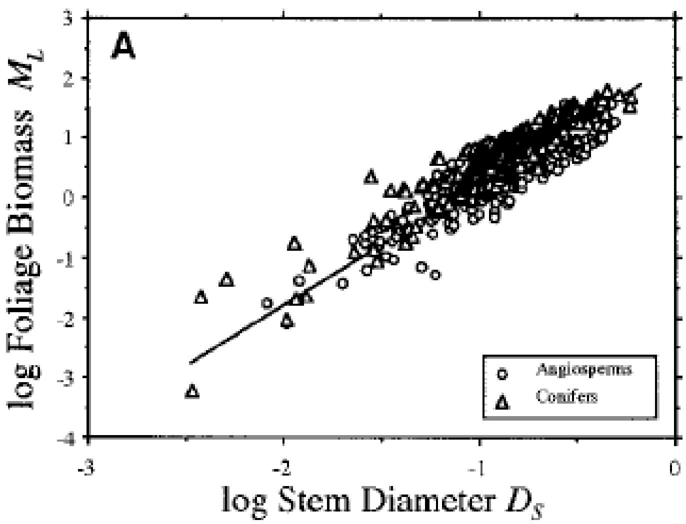
4. Find the following limits if they exist. If they don't exist, state so (8 pts each)

   (a) $\lim_{n\to\infty} \frac{2n-3}{5n+2}$ (b) $\lim_{n\to\infty} \sqrt{\frac{16n^2-n}{n^2+2n}}$

5. Below is Figure 1A from the paper by Enquist and Niklas (Science Feb. 22, 2003) discussed in class. This shows that plant stem diameter D_s (measured in meters) is allometrically related to foliage biomass M_L (measured in kg dry weight). The linear regression through these data is shown, and note that it approximately goes through the points (- 1.5, -1) and (-.5, 1) on this log-log graph.

(a) Give an equation which expresses M_L as a function of D_s being sure to estimate any parameters in the equation using the regression line. So you should have an answer with M_L on the left hand side of the equation by itself.(12 pts)

(b) If you compare two plants, with plant A having twice the stem diameter of plant B, how are the foliage biomasses of plant A and plant B related? (4 pts.)



6. Carry out the following calculations where possible. If the calculation is not defined, clearly state that it is not defined. (8 pts each)

(a) $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 3 & 4 \\ 6 & -2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 2 & 0 & -2 \\ -1 & 4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$

7. Among patients complaining of sudden chest pain, tests indicate that 40% have had a heart attack. One symptom of a heart attack is lightheadedness which occurs in 70% of those who have had a heart attack . For other patients with sudden chest pain, who have not suffered a heart attack, only 20% are lightheaded.

(a) What fraction of all patients with sudden chest pain have lightheadedness? (8 pts)
(b) A patient with sudden chest pain is not lightheaded – what is the probability this patient has had a heart attack? . (8 pts)

8. Suppose that 49% of a population has genotype AA and there are only 3 genotypes (AA, Aa, and aa). If the population is at Hardy-Weinberg equilibrium, (8 pts each part)

(a) what percentage of the population is type Aa ?

(b) what is the gene frequency of the allele a in the population's gametes?

9. A survey of children at a summer camp finds that 30% brush their teeth daily, 60% bathe daily, and 20% both bathe and brush their teeth daily. Find the probability a randomly selected child at this camp (7 pts each part)

(a) brushes their teeth daily or bathes daily or both

(b) brushes their teeth daily or bathes daily but not both

(c) neither brushes their teeth daily nor bathes daily

10. Suppose a population consists of juveniles and adults, and individuals live for one year in each of these states. Juvenile females produce on average 2 new juvenile female offspring each year, while adults produce on average 9 juvenile female offspring before the adult dies. On average, only one third of all juveniles survive to become adults. (6 pts each part)

(a) Give a matrix equation that allows you to project the number of juvenile and adult females at any year n, given that you know the number of these at time 0. That is write an equation that allows you to get J(n) and A(n) given J(0) and A(0).

(b) Suppose there are 10 juvenile females and one adult female present at time 0. How many of each will be present two years later (that is, find J(2) and A(2))?

(c) If the population were to exist for a long time, what would its long term growth rate be?

(d) What would be the long-term ratio of juveniles to adults in the population?

Have a great Holiday!!