MATH 151 - FALL 2004

Answers to Sample Final Exam:

1. (a) log R = a log W + log b where a = 1.8 and log b = 34.
(b) $R = 34\ W^{1.8}$
(c) $2^{1.8} = 3.5$

2. $\lambda = 2$ has eigenvector $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\lambda = 4$ has eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

3. (a) $\begin{bmatrix} 8 & 16 & 25 \\ 0 & 5 & 6 \\ 2 & 9 & 7 \end{bmatrix}$
(b) $\begin{bmatrix} 9 \\ 11 \end{bmatrix}$
(c) not defined

4. (a) $\begin{bmatrix} J_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ .25 & 0 \end{bmatrix} \begin{bmatrix} J_t \\ A_t \end{bmatrix}$ for the case in which you assume the sex ratio is 50:50 and females produce booth male and female offspring. If you assume all 8 offspring are female, then the matrix becomes $\begin{bmatrix} 0 & 8 \\ .25 & 0 \end{bmatrix}$.
(b) $\begin{bmatrix} 12 \\ 1 \end{bmatrix}$ so it returns to the same in two time periods. If you use the second matrix above (with an 8 rather than a 4) you get $\begin{bmatrix} 24 \\ 2 \end{bmatrix}$
(c) The eigenvalues are 1 and -1 for the first case, so long term growth rate is 1, but this means the population actually oscillates every two time periods returning to the same structure, with a ratio of 12:1 juveniles to adults as at the start. For the second matrix, the eigenvalues are $\sqrt{2}$and $-\sqrt{2}$ and the long term ratio of juveniles to adults is $4\sqrt{2}$:1.

5. (a) .9 (b) .6 (c) .1

6. (a) 1/16 (b) 1/28

7. (a) .52 (b) 1/13 (c) 1/3

8. (a) $x_n = (3.5)\ 3^n + .5$
(b) $x_n = 5n + 2$
(c) $x_n = (2)\ 3^n - 4\ (-1)^n + 4$
(d) $x_n = (2)\ 3^n - 2^n + 4$

9. 1-P[No 5's at all] = 1 - 125/216 = .422

10. (a) $x_n = x_0\,(1.3)^n$
(b) sometime during the 4th day - solving gives n = 4.19, so if you check only daily, the tripling will be observed on day 5.

11. (a) 1/3 (b) 1/16 (c) not defined (d) $\sqrt{3}$