Mathematical modeling of immune system development: connections to body mass growth and metabolic rate

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Abstract
To describe immune system development, a theoretical approach is considered based on the assumption about the availability of the immune system goal-seeking behavior – physiological adaptation. For this, we consider an extended mathematical model of age-related changes in population of peripheral T cells (Romanyukha, Yashin, 2003). The energy cost of antigen burden is estimated and used as a measure of the immune system effectiveness. Our treatise is based on the assumption of linear dependence of antigen load from basal metabolic rate, which, in turn, depends on body mass following the allometric relationship – 3/4 power scaling law (Kleiber, 1932; West, Brown, 2005). The dependence of optimal resource allocation in the immune system from the parameters of antigen load is studied.

Mathematical model
To describe the dynamics of age-related changes in population of peripheral T cells, the following model system was considered:

\[
\begin{align*}
\frac{dN}{dt} &= \frac{\alpha - \beta N c}{\gamma + \delta N} - \lambda_1 N - \lambda_2 N M \\
\frac{dM}{dt} &= \lambda_1 N - \lambda_2 N M
\end{align*}
\]

The model variables depend on age \(t\): \(N\) – the rate of naive T cells influx from thymus into the immune system; \(M\) – the concentration of naive T cells in IPLT, \(L\) – the length of telomeres in naive T cells leaving thymus, \(P\) – the length of telomeres in naive T cells, \(P^*\) – the length of telomeres in memory T cells, \(V\) – the volume of IPLT, \(m\) – the body mass.

Parameters’ estimation
1. Simple data fit: logarithmic least-squares
\[
F = \sum \left(\log \left(\frac{X}{x_j}\right)\right) \rightarrow \min
\]

\(X\) - data of observations; \(x_j\) – solution to the model system.

2. Strong immune system or effective reproduction? - the principle of minimal energy dissipation
\[
W = W_0 \rightarrow \min
\]

Energy penalty of the immune system function (power units) – maintenance and production of immune cells

Theoretical prediction for body mass growth of multicellular organisms (West, Brown, 2005)

Body mass can be used as a surrogate measure of antigen load: \(L = \frac{\alpha}{\beta M}\)

Results
1. Simple data fit: logarithmic least-squares
2. Strong immune system or effective reproduction? - the principle of minimal energy dissipation

Optimization technique
Differential evolution (DE) algorithm (Storn, Price, 1997)

For further information
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