The tubuloglomerular feedback (TGF) system in the kidney is a key regulator of filtration rate, which has been shown in physiologic experiments in rats to mediate oscillations in tubular fluid pressure and flow, and in NaCl concentration in tubular fluid of the loop of Henle. In this study, we developed a mathematical model of the TGF system that represents NaCl transport along a short loop of Henle with compliant walls. The proximal tubule and the outer-segment segment of the descending limb are assumed to be highly permeable and have active NaCl transport. A bifurcation analysis of the TGF model equations was performed by deriving and finding roots of the characteristic equation, which arises from a linearization of the model equations. The analysis revealed a complex parameter region that allows a variety of qualitatively different model equations: a regime having one stable, time-independent steady-state solution; regimes having one stable oscillatory solution only; and regimes having multiple possible stable oscillatory solutions. Model results suggest that the compliance of the proximal tubule, descending limb, and thick ascending limb walls increases the tendency of the model TGF system to oscillate, consistent with previous modeling results.

Our previous mathematical model of the TGF consists of simple components; the thick ascending limb (TAL) is represented by a rigid tube with plug flow that carries only the NaCl ion, and the actions of proximal tubule and descending limb were modeled by a linear function that represents glomerular-tubular balance in proximal tubule and water absorption from descending limb.

In this study, we aim to investigate the role of the proximal tubule and descending limb on TGF dynamics, and to assess the extent to which the high degree of nonlinearity exhibited by our model may be an artifact of the rigid tube formulation. We developed a TGF model that represent a short loop of Henle having pressure-driven flow and compliant walls, and we analyzed the model by means of linearization and numerical computations.

Abstract

The tubuloglomerular feedback (TGF) system in the kidney, which is a key regulator of filtration rate, has been shown in physiologic experiments in rats to mediate oscillations in tubular fluid pressure and flow, and in NaCl concentration in tubular fluid of the loop of Henle. In this study, we developed a mathematical model of the TGF system that represents NaCl transport along a short loop of Henle with compliant walls. The proximal tubule and the outer-segment segment of the descending limb are assumed to be highly permeable and have active NaCl transport. A bifurcation analysis of the TGF model equations was performed by deriving and finding roots of the characteristic equation, which arises from a linearization of the model equations. The analysis revealed a complex parameter region that allows a variety of qualitatively different model equations: a regime having one stable, time-independent steady-state solution; regimes having one stable oscillatory solution only; and regimes having multiple possible stable oscillatory solutions. Model results suggest that the compliance of the proximal tubule, descending limb, and thick ascending limb walls increases the tendency of the model TGF system to oscillate, consistent with previous modeling results.

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