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HOW TO INTERACT TODAY
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Objectives of the Mathematics of Gun Violence Workshop

- Review the existing approaches on the mathematics and modeling of gun violence
- Identify and prioritize areas in the field that require further research
- Develop cross-disciplinary collaborations to gain new perspectives
- Suggest research and data-collection that could assist evidence-based policy recommendations
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Webinar Objectives

- **Terminology**
  - The process of modeling
  - Mathematical models
  - Statistical models
  - Gun crime

- **Existing Gun Crime and Gun Violence Models**
  - Analyzing spatio-temporal distribution of gun crime and gun violence
  - Impacts of constraining gun availability on crime and violence
  - Effects of population characteristics on crime and violence
  - Intervention attempts and their impacts

- **Questions**
Terminology
Objectives of Models

- Provide a framework to assemble bodies of facts/observations (standardize data collection)
- Clarify hypotheses and chains of argument
- Identify key components in systems
- Allow investigation while accounting for societal or ethical constraints
- Provide the ability to consider spatial and temporal change simultaneously
- Prompt tentative and testable hypotheses
- Serve as a guide to decision making in circumstances where action cannot wait for detailed studies
- Provide a means to look at general patterns and trends
- Predict how a system will behave under different management, and control the system to meet some objective
Models and tradeoffs

No one model can do everything!
Model evaluation

- Given the many objectives for models, we should expect there to be multiple criteria for determining whether a model is useful.
- Before developing a model, criteria should be established for evaluating its use.
- Evaluation procedures should account for constraints of data, effort, resources, and computation.
- Evaluation criteria should be taken into consideration when assessing methods, level of detail, scale, and what can be ignored when deciding on a model.
Mathematical models

Elucidate key features of a system and ignore what is not relevant to the specific question to be answered

Smith? 2008, Yong et al. 2015
Statistical models

Methods of determining the properties of a given system in terms of relationships between system components

https://bolt.mph.ufl.edu/6050-6052/unit-4/
Types of Models

- Discrete
- Continuous
- Dynamic
- Static
- Deterministic
- Stochastic
Types of Models

- **Deterministic** - assumes the outcomes are completely determined by the model
- **Stochastic** - Focuses on the probability distributions of various outcomes in the system
- **Continuous** - allows variables to change at any point in time
- **Discrete** - tracks changes to variables in discrete time steps
- **Dynamic** - describes how a system changes over time
- **Static** - describes the state of a system at one point in time

Otto and Day 2011
Examples of Mathematical Models

- Deterministic, dynamic, continuous
  - Example: Ordinary differential equation (ODE) model of infectious disease spread (Kermack-McKendrick)

- Stochastic, dynamic, discrete
  - Example: Markov model of sentence structure

http://www.public.asu.edu/~hnesse/classes/sir.html

https://hackernoon.com/from-what-is-a-markov-model-to-here-is-how-markov-models-work-1ac5f4629b71
Types of Statistical Models

- **Regression** - Method to determine relationships between two or more variables
- **Time Series Analysis** - Analyzes patterns in a sequence of observations over time
- **Markov Models** - Used to explain cases in which the future of the process depends only on the present, not the past
- **Spatial Modeling** - assesses the relationship between events and their spatial distribution
- **Multivariate Statistics** - considers simultaneous interactions between a number of factors

Bennyan et al., 2003, Box et al., 2015, Draper and Smith 2014, Ripley 2004, Tinsley and Brown 2000, Unkel et al., 2011
Examples of Statistical Models

- Regression
- Example: ice cream and drowning

- Spatial Modeling
- Species distribution of endangered crayfish


- 467,321 persons were victims of a crime committed with a firearm
- Firearm crimes comprised 8% of all violent crimes (rape, assault, etc.)
- There were 11,101 firearm homicides
- Firearms were used in:
  - 68% of murders
  - 41% of robbery offenses
  - 21% of aggravated assaults

Gun violence research - some history

- (1993) Kellerman et al. publish *Gun ownership as a risk factor for homicide in the home*
- (1996) Dickey Amendment: “none of the funds made available for injury prevention and control at the Centers for Disease Control and Prevention (CDC) may be used to advocate or promote gun control”
- (2012) Extended to also cover the National Institutes of Health (NIH)
- (2017) Gun violence is the least-researched cause of death and the second least-funded cause of death after falls

Existing Gun Violence and Gun Crime Models
Analyzing Spatio-temporal Distribution of Gun Violence and Gun Crime
Spatial Statistics Approaches

● Method: Distance matrices and $K$-functions for spatial dependence
  ○ Question of interest: Is there a concentration of shootings near schools?
  ○ Reference: Barboza 2018

● Method: Bayesian spatio-temporal point process
  ○ Question of interest: Distinguish between clustered but non-diffusing gun violence and clustered gun violence resulting from diffusion
  ○ Reference: Loeffler and Flaxman 2017

● Method: Network-based computation methods
  ○ Question of interest: Estimate the strength and extent of the spatial influence of physical features on gun violence
  ○ Reference: Xu and Griffiths 2017

● Method: Cluster detection
  ○ Question of interest: Investigate whether different homicide types have different patterns of clustering and movement
  ○ References: Zeoli et al., 2015
Methods of Hot Spot Analysis

- Methods:
  - Marked point process
  - Growth curve regression

- Questions of interest:
  - Show how point process models of crime can be extended to capture both short-term and long-term patterns of risk
  - Uncover distinctive developmental trends in gun assault incidents at street segments and intersections

\[ \lambda(x, y, t) = \mu(x, y) + \sum_{t > t_i} g(x - x_i, y - y_i, t - t_i, M_i). \] (6)

Mohler 2014, Braga et al., 2010

Fig. 5: The spatial distribution of micro places with stable and volatile concentrations of serious gun violence in Boston.
Other Methods of Cluster Detection and Analysis

- **Method:** Risk Terrain Modeling (RTM) with environmental risk factors
- **Question of interest:** Investigate the application of RTM to forecast gang violence and predict future gang assaults and gang homicides
- **Reference:** Valasik et al., 2018
Comparing Hot Spots to RTM

- Objective: Assess the possible differences in the accuracy and precision of two methodological mapping techniques as predictors of gun crime
- Method: RTM, Nearest Neighbor Hierarchical (Nnh)
Impacts of Constraining Gun Availability on Violence and Crime
Mathematical Modeling of Gun Constraints

- **Method: Game Theory**
  - Question of interest: Construct a simple victim-criminal interaction model to provide insights about the desirability of gun crime legislation
  - Reference: Taylor 1995

- **Method: Game Theory and Probabilistic Analysis**
  - Question of interest: Present a mathematical framework to analyze the debate about gun control
  - Reference: Wodarz and Komarova 2013
Statistical Analysis of Gun Constraints

- **Regression**
  - Two stage least-squares
    - Question of interest: Determine the effects that gun control restrictions and gun prevalence have on rates of violence and crime
    - Reference: Kleck and Patterson 1993
  - Limited information maximum likelihood
    - Question of interest: Examine the relationship between gun availability and crime in a cross-national sample of cities
    - Reference: Altheimer 2010

- **Cross-sectional Time Series**
  - Question of interest: Determine whether restrictiveness-permissiveness of state gun laws or gun ownership are associated with mass shootings
  - Reference: Reeping et al. 2019
Effects of Population Characteristics on Violence and Crime
Modeling the Effects of Population Characteristics

- **Method: Probabilistic Contagion Model**
  - Question of interest: Evaluate the ability of a social network epidemic model to predict who will become a victim of gun violence
  - Reference: Green et al., 2017

- **Method: Network Modeling**
  - Question of interest: Examine the role of neighborhood-level criminal networks in shaping the distribution of crime throughout cities
  - Reference: Bastomski et al., 2017
Statistical Analysis of Population Characteristics

- **Method: Multivariate Statistical Methods**
  - Question of interest: Examine the epidemiology of nonfatal firearm violence on the Westside of Chicago
  - Question of interest: Assess the relationship between perceived collective efficacy, its subscales of social cohesion and informal social control, and exposure to gun violence
  - Reference: Fitzpatrick et al. 2018, Riley et al. 2017

- **Method: Trend analysis, regression**
  - Question of interest: Examine Chicago crime data to see if there is evidence of a “Ferguson Effect” and to see if the availability of illegal firearms can explain the violence rise in Chicago
  - Question of interest: Examine the relationships between isolated youth, illegal gun availability, structural disadvantage, and Southern culture with gun crime
  - Reference: Towers and White 2017, Dierenfeldt et al. 2017

- **Method: Machine learning and random forest algorithms**
  - Question of interest: Identify an optimal set of predictors for urban interpersonal firearm violence rates using a broad set of community characteristics
  - Reference: Goin et al. 2017
Statistical Analysis of Population Characteristics

- **Objective:** Explore whether or not contagion is evident in more high-profile incidents, such as school shootings and mass killings
- **Method:** Contagion model
- **Results:** Mass killings involving firearms are incited by similar events in the immediate past, as are school shootings

![Figure 2. Relationship of state prevalence of firearm ownership, mental illness, and state rankings of strength of firearm legislation, to the state incidence of mass killings, school shootings, and mass shootings.](image)
Intervention Attempts and their Impacts
Modeling Intervention Attempts and their Impacts

- **Method:** Ordinary Differential Equations
  - Question of interest: Introduce a Susceptible-Transmitter-Victim epidemic model to explore the impact of violence interruption on the spread of violence

![Flow diagram for STV model](image-url)
Statistical Analysis of Intervention Attempts and their Impacts

- **Method: Autoregressive Integrated Moving Average (ARIMA) Models**
  - Question of interest: Test the impacts of Project Longevity on group-involved shootings and homicides in New Haven, Connecticut
  - Question of interest: Identify the impact of the Strategic Subjects List (SSL) pilot on individual- and city-level gun violence and to test possible drivers of results

- **Method: Multivariate and exploratory structural equation modeling**
  - Question of interest: Measure perceived norms and viewpoints regarding gun violence in response to implementing the Safe Streets intervention in Baltimore, Maryland
  - Reference: Milam et al. 2017

- **Method: Bayesian hierarchical models**
  - Question of interest: Test the relationship between neighborhood misdemeanor policing and homicide
  - Reference: Cerda et al. 2009
Conclusions
Conclusions

- There is limited published research on the mathematics of gun violence and gun crime
- The majority of models are statistical in nature, with most using regression methods
- A number of models have analyzed the spatio-temporal spread of crime and observed the effects of population characteristics
- Fewer published studies observe the impacts of constraining gun availability and intervention attempts
Other Mathematical Approaches to Gun Violence and Gun Crime

- Difference Equations
- Partial Differential Equations (PDEs)
- Cellular Automata
- Agent-Based Models (ABMs)
- Optimal Control Theory
Questions?

Workshop Website:
http://www.nimbios.org/workshops/WS_gunviolence
References

References

References

References

References


Spatial Statistics Approaches

- Question: Is there a concentration of shootings near schools?
- Method: Distance matrices and $K$-functions to determine spatial dependence
- Results: There is non-random clustering of shootings and a greater number of shootings were clustered within short distances of schools than would be expected if there were no spatial dependence

Fig. 10 Relative concentration of shootings and distance from Boston Public School (km)
Spatial Statistics Approaches

- **Objective:** Distinguish between clustered but non-diffusing gun violence and clustered gun violence resulting from diffusion
- **Method:** Bayesian spatio-temporal point process and classic space/time interaction tests
- **Results:** Contemporary urban gun violence does diffuse in both space and time, but only slightly

\[
\lambda(x, y, t) = m_0 \cdot \mu(x, y, t) + \theta \sum_{i \mid t_i < t} \omega \exp(-\omega(t - t_i)) \\
\frac{1}{2\pi\sigma^2} \exp\left(-\left((x - x_i)^2 + (y - y_i)^2\right)/(2\sigma^2)\right)
\]

Loeffler and Flaxman 2017

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Fig. 2 The self-excitatory component of the Hawkes process, corresponding to the product of a spatial and temporal kernel, is visualized in space a at time \( t = 0 \), in time b at distance \( x = 0 \), and in space/time c. In a and b the blue line is the mean of the posterior distribution. The gray lines show samples from the posterior distribution, reflecting the (small amount) of posterior uncertainty in the parameter estimates (Color figure online)
Spatial Statistics Approaches

- **Objective:** Estimate the strength and extent of the spatial influence of physical features on gun violence
- **Method:** Network-based computation methods
- **Results:** Liquor stores, grocery stores, bus stops, and residential foreclosures are shooting attractors in Newark, NJ and the magnitude is strongest in the immediate vicinity of each physical feature

Fig. 1 Network Cross K Function identifying attractors of gun violence in Newark, NJ 2012

Xu and Griffiths 2017
Spatial Statistics Approaches

- Objective: Investigate whether different homicide types have different patterns of clustering and movement
- Methods: Cluster detection
- Results: Gang motivated homicides cluster and diffuse and overlap with revenge and drug-related homicides. Escalating disputes and non-intimate partner familial homicides cluster but do not diffuse. Intimate partner and robbery homicides do not cluster

\[
E[h] = p \times H/P, \quad I() = \left( \frac{h}{E[h]} \right)^e \left( \frac{H - h}{H - E[h]} \right)^{c-e} I()
\]

Zeoli 2015
Hot Spots

● Objective: Show how point process models of crime can be extended to capture both short-term and long-term patterns of risk
● Method: Marked point process
● Results: A marked point process performs better than classic hotspot methods

\[
\lambda(x, y, t) = \mu(x, y) + \sum_{t > t_i} g(x - x_i, y - y_i, t - t_i, M_i).
\]

Mohler 2014
Modeling Gun Constraints

● Objective: Construct a simple victim-criminal interaction model to provide insights about the desirability of gun crime legislation
● Method: Game theory
● Results: Gun control laws that target criminals are more likely to be successful than those that attempt to induce a general scarcity of firearms

Fig. 1. The basic model.

Fig. 3. A more complex model.

Taylor 1995
Modeling Gun Constraints

- **Objective:** Present a mathematical framework to analyze the debate about gun control
- **Method:** Probabilistic and game-theoretic analysis
- **Results:** To minimize the gun-related homicide rate, we must either have a ban on private firearms possession or allow the general population to carry guns

Wodarz and Komarova 2013
Statistical Analysis of Gun Restraints: Regression

- **Objective:** Determine the effects that gun control restrictions and gun prevalence have on rates of violence and crime
- **Method:** Two-stage least-squares regression
- **Results:**
  - Gun prevalence levels generally have no net positive effect on total violence rates
  - Homicide, gun assault, and rape rates increase gun prevalence
  - Gun control restrictions have no net effect on gun prevalence levels
  - Most gun control restrictions generally have no net effect on violence rates
Statistical Analyses of Gun Constraints: Regression

- **Objective:** Examine the relationship between gun availability and crime in a cross-national sample of cities
- **Method:** Limited information maximum least squares regression
- **Results:** Gun availability influences rates of assault, gun assault, robbery, and gun robbery
Statistical Analysis of Gun Constraints: Time Series

- Objective: Determine whether restrictiveness-permissiveness of state gun laws or gun ownership are associated with mass shootings
- Method: Cross-sectional time series
- Results: States with more permissive gun laws and greater gun ownership had greater rates of mass shootings and a growing divide seems to be emerging between restrictive and permissive states

Fig 1 | Rates of mass shootings over time in restrictive versus permissive states for a restrictiveness-permissiveness score of 50 (A) and 79 (B). Years 1998-2014 were included because of the lag of the permissiveness score
Modeling the Effects of Population Characteristics

- **Objective:** Evaluate the ability of a social network epidemic model to predict who will become a victim of gun violence
- **Method:** Probabilistic contagion model
- **Results:** Gun violence follows an epidemic-like process of social contagion that is transmitted through networks

Green 2017
Modeling the Effects of Population Characteristics

● Objective: Examine the role of neighborhood-level criminal networks in shaping the distribution of crime throughout cities

● Methods: Network modeling

● Results: A neighborhood’s embeddedness increases the local homicide rate, even after controlling for other factors

Fig. 1. Network k-core structure. The maximum k-core appears at the center, and consists of 123 neighborhood-nodes (approximately 36 percent of all nodes) that each have a degree of 50 or higher within this highly connected sub-community of nodes.
Statistical Analysis of Population Characteristics

- Objective: Examine the epidemiology of nonfatal firearm violence on the Westside of Chicago
- Method: Pearson’s Chi square, ANOVA
- Results: Nonfatal firearm violence is a major problem on the Westside of Chicago, particularly for young, black men, but the incidence of gun violence has not changed significantly between 2005 and 2016
Statistical Analysis of Population Characteristics

- Objective: Examine Chicago crime data to see if there is evidence of a “Ferguson Effect” and to see if the availability of illegal firearms can explain the violence rise in Chicago
- Method: Trend analysis, regression
- Results: There was no relationship between the timing of the 2014 Ferguson riots and the Chicago crime trends, but there is a significant association between the proliferation of firearms in Chicago and patterns in firearm-related violent crime

Figure 3: Test of the predictive capability of models for the annual number of firearm assaults, regressed on a training sample derived from data from the West Garfield Park community area, and tested with data from the Greater Grand Crossing community area.
Statistical Analysis of Population Characteristics

- Objective: Assess the relationship between perceived collective efficacy, its sub scales of social cohesion and informal social control, and exposure to gun violence
- Method: Chi-squared test and linear regression
- Results: There is an association between perceived collective efficacy and exposure to gun violence, but community members can be activated to prepare themselves for future gun violence events

Riley 2017
Objective: Identify an optimal set of predictors for urban interpersonal firearm violence rates using a broad set of community characteristics

Method: LASSO and random forest algorithm regression trees

Results: A set of 18 predictive covariates explain 77.8% of the variance in firearm violence rates, many of which are related to education and socioeconomic conditions

\[ f(x) = \sum_{m=1}^{M} c_m I(x \in R_m) \]
Statistical Analysis of Population Characteristics

- Objective: Examine the relationships between isolated youth, illegal gun availability, structural disadvantage, and Southern culture with gun crime
- Method: Binomial regression
- Results: Illegal gun availability and structural disadvantage maintain direct relationships with city-level gun crime counts
Modeling Intervention Attempts and their Impacts

- **Objective:** Introduce a Susceptible-Transmitter-Victim epidemic model to explore the impact of violence interruption on the diffusion of violence.
- **Method:** Ordinary Differential Equations
- **Results:** Targeting all potential violence transmitters can reduce gun violence three times more than an intervention that only targets gun-owning individuals. Having individuals in the populations transmitting but not participating in gun violence is sufficient to sustain a gun violence epidemic.

**Fig. 3 Impacts of violence interruption on the incidence of gun assaults.** This figure depicts effects of violence intervention efforts on total incidences of gun assaults (per 100,000 people) per year over 30 years. The **solid, blue line** represents baseline values, \( \gamma = 44.35, q_N = 0.5, q_L = 0.51, q_I = 0.49, \ell_L = 0.05, \ell_I = 0.13, \beta = 2.1, \alpha_L = 0.004, \alpha_I = 0.006, \sigma_N = 1.92, \sigma_L = 1.62, \sigma_I = 0.87, \) where the per capita gun assault rate is 172.17 at 30 years. The **dashed, green line** depicts a 10% increase in violence intervention efforts in the LGO and IGO transmitter populations only (\( \sigma_N = \sigma_L = \sigma_I = 1.9 \) and all other parameters left at their baseline values), resulting in a decrease in the per capita gun assault rate to 76.9 at 30 years. The **dotted, red line** represents a 10% increase from baseline values in violence intervention efforts (\( \sigma_N = 2.11, \sigma_L = 1.78, \sigma_I = 0.96, \) and all other parameters remained fixed at their baseline value) across all transmitter populations. The resulting per capita gun assault rate at 30 years is 55.58.
Statistical Analysis of Intervention Attempts

- Objective: Test the impacts of Project Longevity on group-involved shootings and homicides in New Haven, Connecticut
- Method: Autoregressive Integrated Moving Average (ARIMA) Models
- Results: Project Longevity is associated with a reduction of almost five group-member involved incidents per month

Figure 2. Monthly distribution of the shootings and homicides in New Haven and in Hartford before and during Project Longevity.
Statistical Analysis of Intervention Attempts

- Objective: Identify the impact of the Strategic Subjects List (SSL) pilot on individual- and city-level gun violence and to test possible drivers of results
- Method: Autoregressive Integrated Moving Average (ARIMA) models, mediation analysis
- Results: Individuals on the SSL are not more or less likely to become a victim of a homicide or shooting than the comparison group, but the treated group is more likely to be arrested for a shooting

Saunders 2016
Statistical Analysis of Intervention Attempts

- Objective: Measure perceived norms and viewpoints regarding gun violence in response to implementing the Safe Streets intervention in Baltimore, Maryland
- Methods: Chi-squared tests and exploratory structural equation modeling (ESEM)
- Results: There was greater improvement in attitudes towards violence in the intervention community following the implementation of the Safe Streets program

Milam 2017
Statistical Analysis of Intervention Attempts

- **Objective:** Test the relationship between neighborhood misdemeanor policing and homicide
- **Method:** Bayesian hierarchical models
- **Results:** Misdemeanor policing reduces homicide rates, but physical disorder is not a mediator of the impact of such policing

\[
\Delta Y_{it} = \alpha_i + \beta_1 \Delta X_{arrests_{it}} + \beta_2 \Delta X_{publicassist_{it}} + \beta_3 \Delta X_{order_{it}} + \mathbf{X} \mathbf{B}_i + \lambda_i
\]

**Figure 2.** Average gun-related homicide rate per 100,000 population by police precinct, New York City; A, 1990–1994 and B, 1995–1999.

Cerda 2009