Example 4.8 (Elephants)

- It has been determined that for any elephant, surface area of the body can be estimated as an allometric function of trunk length:

$$surface\ area = (some\ number) \times (trunk\ length)^{(some\ number)}$$

- For African elephants the allometric exponent is 0.74:

$$y = ax^{0.74}$$

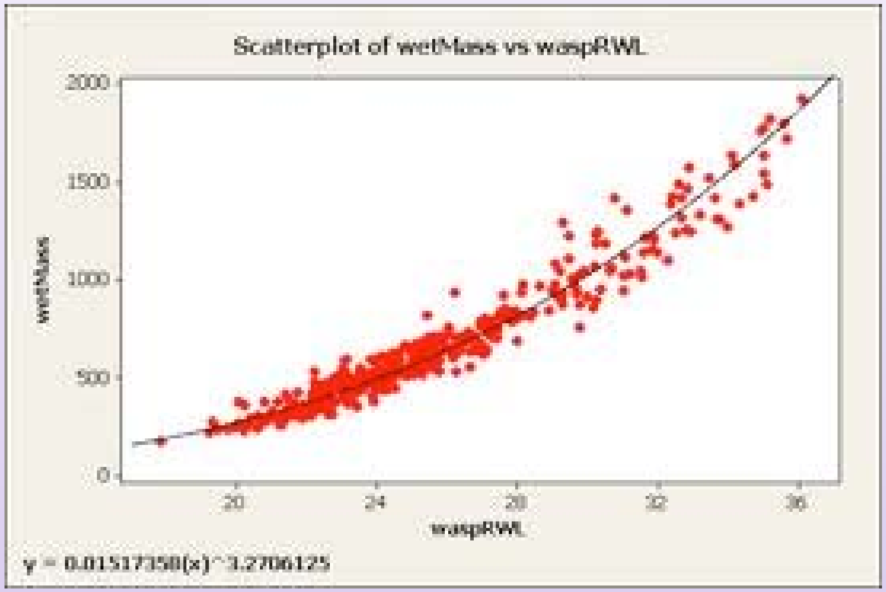
  y = surface area

  x = truck length

- If a particular elephant has a surface area of 200 ft^2 and a trunk length of 6 ft, what is the expected surface area of an elephant with a trunk length of 7 ft?

Overview

Mass Vs. Right Wing Length
Non-Linear Model for Wasps



- As pointed out above, data often appear to have a strong relationship, but that relationship is not linear
- We would like to apply the same analysis as before, but we would need to develop a new method to get the best fit curve
- In some situations, however, a *rescaling* of the data could transform it in such a way that the new relationship is linear

Rewriting Equations

- We start with an exponential equation:

$$y = a \cdot c^x$$

$$\ln(y) = \ln\left(a \cdot c^x\right)$$

$$\ln(y) = \ln a + \ln c^x$$

$$\ln(y) = \ln a + x \ln c$$

$$\ln(y) = (\ln c)x + \ln a$$

Rewriting Equations

$$\ln(y) = (\ln c)x + \ln a$$

$$\text{dependent variable} = \text{constant} \times \text{independent variable} + \text{constant}$$

$$Y = mx + b$$

The new equation is a linear equation with

Y = ln (y), m = ln c, b= ln a

4. (4.4) Rescaling data: Log-Log & Semi-Log Graphs

Rewriting Equations

- Now consider an allometric equation:

$$y = a \cdot x^c$$

$$\ln(y) = \ln\left(ax^c\right)$$

$$\ln(y) = \ln a + \ln x^c$$

$$\ln(y) = \ln a + c \ln x$$

$$\ln(y) = c \ln x + \ln a$$

- Again, we obtain a linear equation with Y = ln (y) and
- X =ln(x) … line Y = c X + ln(a)

Rescaling Data

Exponential	**Allometric**
$y = a \cdot c^x$	$y = a \cdot x^c$
$\ln(y) = (\ln c)x + \ln a$	$\ln(y) = c \ln x + \ln a$

- If we have data that are exponentially related, we rescale the y coordinates of the data by taking their logarithm (x,y) ➔ (x, lny), and then the scatter plot of the rescaled data will be linear
- If we have data that are allometrically related, we rescale the x and y coordinates of the data by taking their logarithm (lnx, lny), and then the scatter plot of the rescaled data will be linear

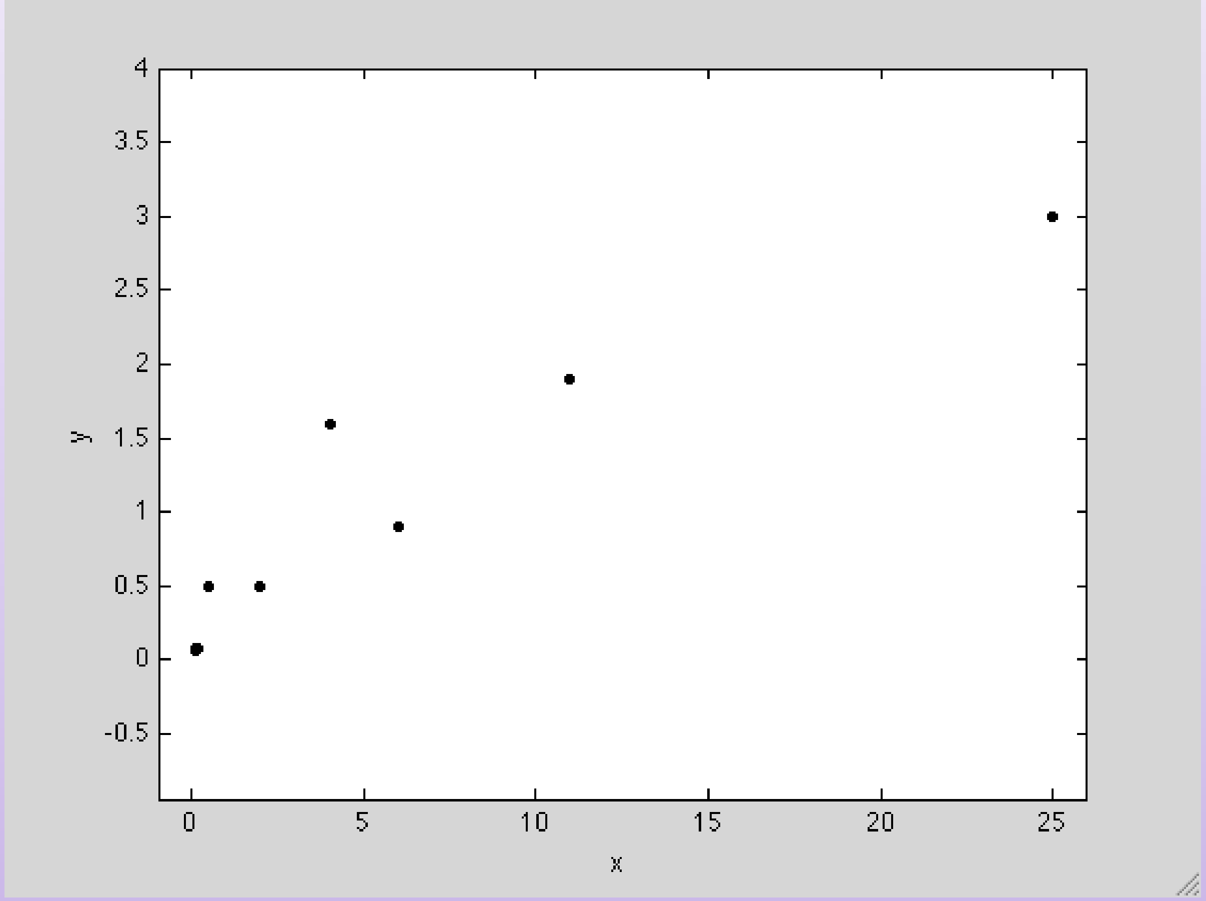
Example 4.11 (Mutation Rates)

- Researchers studying the relationship between the generation time of a species and the mutation rate for genes that cause deleterious effects gathered the following data:

Species	Generation Time (in years)	Genomic Mutation Rate (per generation)
D. melanogaster/D. pseudoobscura	0.1	0.070
D. melanogaster/D. simulans	0.1	0.058
D. picticornis/D. silvestris	0.2	0.071
Mouse/rat	0.5	0.50
Chicken/old world quail	2	0.49
Dog/cat	4	1.6
Sheep/cow	6	0.90
Macaque/New World Monkey	11	1.9
Human/chimpanzee	25	3.0

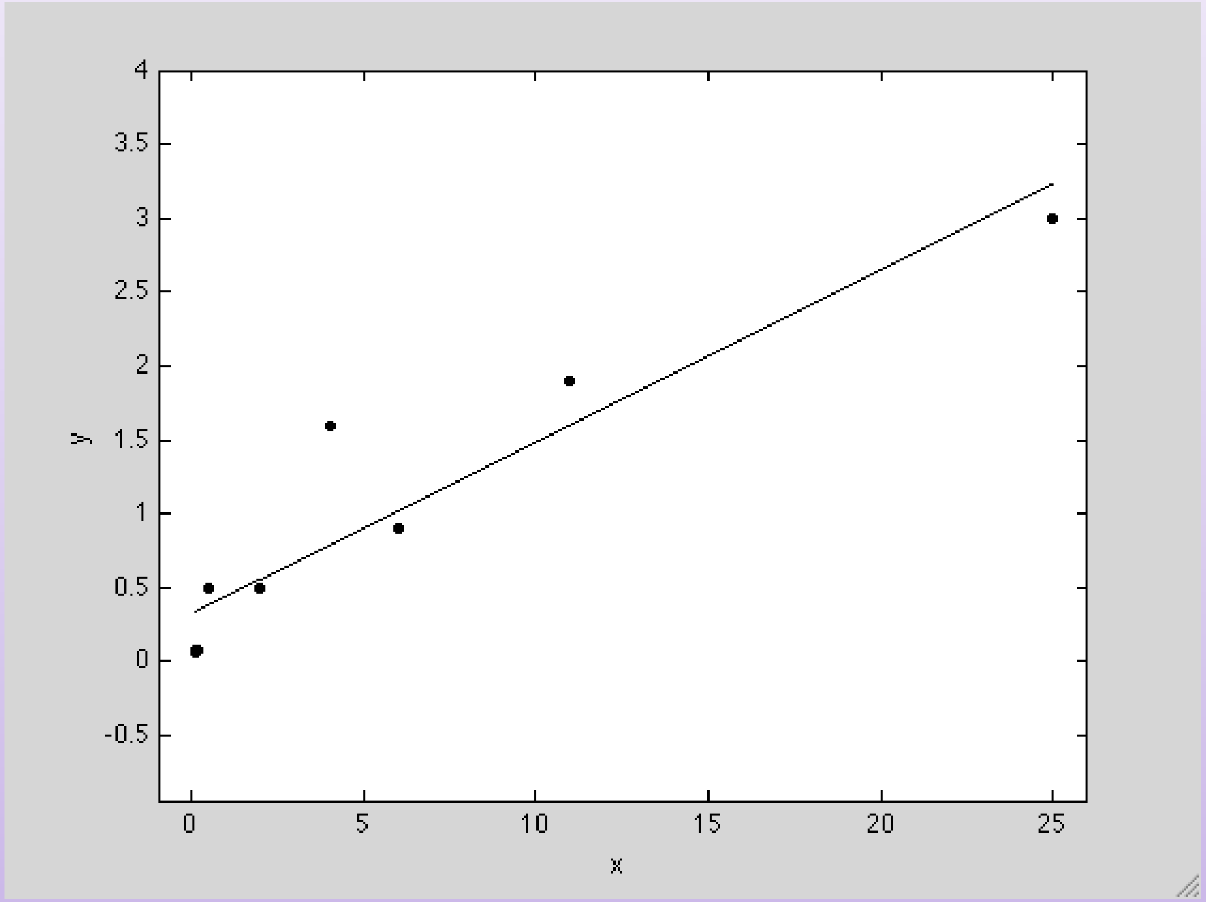
Example 4.11 (Mutation Rates)

- A scatterplot of the data:



Example 4.11 (Mutation Rates)

- A scatterplot of the data with LSR:



Example 4.11 (Mutation Rates)

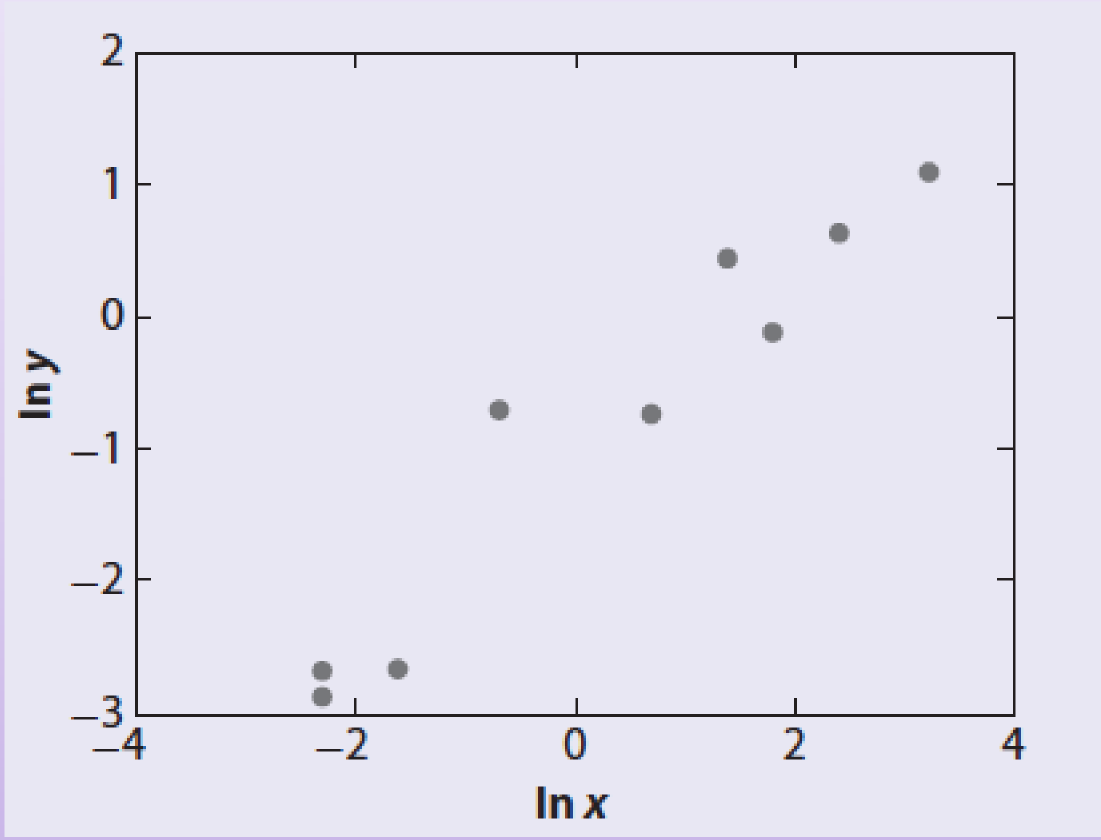
- Is this a good fit?
- The MATLAB output:
  - Eqn for LSR: y = 0.116079 x + 0.323640
  - rho = 0.934107 … R^2 =0.8726
  - The regression line accounts for 87.26% of the variance in the data.
- Suppose we rescale the data:

log - log rescale
⇒
allometric

$\ln x$	$\ln y$
-2.3	-2.659
-2.3	-2.847
-1.6	-2.645
-0.7	-0.693
0.7	-0.713
1.4	0.470
1.8	-0.105
2.4	0.642
3.2	1.099

Example 4.11 (Mutation Rates)

- A scatterplot of the transformed data: LOG-LOG plot



Example 4.11 (Mutation Rates)

- Is this a good fit?
- The MATLAB output:
  - Eqn for LSR: ln y = 0.709705 ln x + -1.031581
  - rho = 0.962501
  - The regression line accounts for 92.64% of the variance in the data
- Which model should we choose?

- Log-log

Example 4.11 (Mutation Rates)

- If we choose the allometric model, we need to solve for y in terms of x:

$$\ln y = 0.7097 \ln x - 1.0316$$

$$e^{\ln y} = e^{0.7097 \ln x - 1.0316}$$

$$y = e^{\ln x^{0.7097}} e^{-1.0316}$$

$$y = x^{0.7097} e^{-1.0316}$$

$$y = 0.3564\, x^{0.7097}$$

- Now we can use the model to predict. Suppose we know a certain species has a generation time of 10 years, we could interpolate the genomic mutation rate of this species:

$$y = 0.3564(10)^{0.7097} \approx 1.8268$$ mutations per generation