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A Finite Element method for the simulation of the lamellipodium of living cells

This is joint work with Dietmar Oelz, RICAM Vienna, and Christian Schmeiser, University of Vienna

Introduction. We present in this talk a Finite Element numerical method that we have developed for the simulation of the both rationally symmetric lammelipodia, as well as lammelipodia that move under the influence of either external forces or variable polymerization.

Background material. The mathematical modeling of the filament dynamics and lammelipodium started in [1] and [2] by Schmeiser, Ölz, and Small. the model incorporates essential mechanical ingredients of the lamellipodium, such as polymerization and depolymerization, adhesion with the substrate, crosslinking between the filaments, and contractile forces caused by myosin. The system that stems from this modeling approach (see also [1]) reads as follows

$$\underbrace{\mu^B \partial_s^2\left(\eta^{\pm}\partial_s^2 F^{\pm}\right)}_{\text{bending}} - \underbrace{\partial_s\left(\eta^{\pm}\lambda^{\pm}\partial_s F^{\pm}\right)}_{\text{in-extensibility}} + \underbrace{\eta^{\pm}\mu^A D_t^{\pm} F^{\pm}}_{\text{adhesion}}$$
$$\underbrace{\pm\partial_s\left(\eta^{+}\eta^{-}\mu_{\pm}^{T}(\varphi-\varphi_0)\partial_s F^{\pm\perp}\right)}_{\text{twisting}} \underbrace{\pm\eta^{+}\eta^{-}\mu^S\left(D_t^{+}F^{+} - D_t^{-}F^{-}\right)}_{\text{stretching}} = 0. \tag{1}$$

The exponents $\pm$ denote the two families of filaments, $F^{\pm}(s,\alpha,t)\in\mathbb{R}^2$, $s\in[0,L]$, $\alpha\in[0,2\pi]$, describes the position of the filament α; $\eta^{\pm}$ are filament length distributions, ϕ_0 is the preferred angle of the crosslinked filaments, and μ^B, μ^A, μ^T, μ^S are state parameters of the problem. $D_t=\partial_t - v\partial_s$ is the material derivative with v the polymerization speed. The system is complemented by initial and boundary conditions.

Numerical method. We focus on this talk in presenting the basic characteristics of the FEM we have developed for the numerical treatment of (1).
We discretize with respect to $(s,a)\in[0,L]\times[0,2\pi]$ and $t>0$. For the numerical approximations we have used two dimensional Finite Element method with Hermite basis function along the s-direction, and Lagrange basis along the a-direction.
The non-linearity in the in-extensibility term is treated by an implicit-explicit in time discretization; this gives rise to two more equations for $\lambda\pm$. The adhesion term is discretized explicitly in time. The stretching and twisting terms couple the two families; the temporal derivatives in the stretching term are treated by a predictor-corrector step.
We refer to the links, below for two numerical tests. In the first we consider a rotational symmetric lamellipodium, and the second one is the twisting term has been deactivated.

- http://homepage.univie.ac.at/nikolaos.sfakianakis/files/LamEquilibrium.mp4
- http://homepage.univie.ac.at/nikolaos.sfakianakis/files/LamZeroTwisting.mp4

References

[1] D.Ölz, Ch. Schmeiser, J. V. Small, *Modelling of the Actin-cytoskeleton in symmetric lamellipodial fragments*, Cell Adhesion and Migration, (2008), 117 - 126

[2] D. Ölz, Ch. Schmeiser *How do cells move? mathematical modelling of cytoskeleton dynamics and cell migration*, Book chapter in *Cell mechanics: from single scale-based models to multiscale modelling*, ed A. Chauviere, L. Preziosi, C. Verdier, (Chapman and Hall / CRC press 2009)

[3] J.V. Small, T., Stradal, E., Vignal, K., Rottner, *The lamellipodium: where motility begins*, Trends Cell Biol. **12** (2002), 112–20