Heinrich Freistühler, University of Constance, Konstanz, Germany
Jan Fuhrmann, University of Heidelberg, Heidelberg, Germany
Angela Stevens, University of Münster, Münster, Germany

Emerging Polymerization Fronts in a Minimal Cytoskeleton Model

The directed motion of eucaryotic cells plays a major role in biological processes as diverse as wound healing, cancer metastasis, immune response, the hunting behavior of different bacteria, and many more. The common driving force of these forms of cell motility is the permanent reorganization and filament turnover of the actin cytoskeleton.
In this presentation we will derive a minimal model for the cytoskeleton of a potentially motile cell being at rest and turned into motion upon some stimulus. The model is comprised of four hyperbolic partial differential equations describing the densities of actin filament tips and one parabolic equation for the actin monomer concentration. These are coupled via the polymerization and depolymerization of monomers at the filamend ends.
We will deduce a free boundary problem for this system where the moving boundary represents the cell membrane which is supported by actin filaments and may be displaced upon their growth or shrinkage.
For this problem we show existence and uniqueness of solutions for small times. Remarkably we cannot easily rule out the emergence of Dirac measure type solutions to the hyperbolic equations together with discontinuities in the monomer concentration – a phenomenon known as interior gradient blow-up.
In fact, the investigation of the hyperbolic limit system without diffusion of monomers reveals possible discontinuous solutions which can be found analytically. Numerical results suggest that at least the local concentration of large numbers of filament ends also occur in the original model with diffusion.
We furthermore find some examples of another type of shock like solutions to the full system with diffusion which are characterized by measure valued filament end densities as well but without exhibiting discontinuities in the monomer concentration.
Morover, we shall provide some numerical results illustrating the evolution of these shocks and comment on their interpretation as moving fronts of polymerizing actin filaments.
Finally, we will also show some simulations where cells are turned into movement with or without the emergence of sharp fronts depending on the particular parameter settings.