Math 152 – Sample Exam 3 – Spring 2016 – Louis Gross

It would be best if you try to take this Sample Exam as if you were sitting in class, using only a calculator. For the actual in-class exam, there will be blank sheets of paper handed out for you to give the answers but it will be important for you to SHOW YOUR WORK even if you are certain your answer is correct. Note that this Sample Exam is designed to take one hour, approximately the same length as the actual exam will be. On the actual exam the formulas for derivatives and antiderivatives below will be provided, but I expect that you will know all the other rules for finding derivatives and antiderivatives. This exam covers Chapters 21-24 but relies on your knowledge of previous chapters so be sure to go over the previous exams.

Table of Derivatives and Antiderivatives:

$D_x(mx+b)=m \qquad D_x(Ce^{kx})=Cke^{kx} \qquad D_x(Ca^x)=\ln(a)Ca^x$

$D_x(\sin(Cx))=C\cos(Cx) \qquad D_x(x^n)=nx^{n-1}\, for\ n\neq -1 \qquad D_x(\ln(x))=\frac{1}{x}$

$D_x(\cos(Cx))=-C\sin(Cx)$

$\int x^n dx=\frac{1}{n+1}x^n+c\ for\ n\neq -1 \qquad \int \frac{1}{x}dx=\ln(x)+c \qquad \int \sin(x)dx=-\cos(x)+c$

$\int \cos(x)dx=\sin(x)+c \qquad \int e^{ax}dx=\frac{1}{a}e^{ax}+c \qquad \int a^x dx=\frac{a^x}{\ln(a)}+c$

$\int \frac{f'(x)}{f(x)}dx=\ln(f(x))+c \qquad \int f'(g(x))g'(x)dx=f(g(x))+c$

$\int f'(x)e^{f(x)}dx=e^{f(x)}+c \qquad \int f'(x)f(x)^n dx=\frac{1}{n+1}f(x)^{n+1}+c$

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1. The zooplankton density in a lake at noon is found to be reasonably well described by an exponential distribution since most of the zooplankton migrate towards the surface at mid-day. The density function at midnight is $\rho_M(x)=1000e^{-x/2}$ where x is the depth and $\rho_M(x)$ is the number of zooplankton per m^3 in a standard 1 m^2 water column. At noon, the density function is $\rho_N(x)=1000xe^{-x/2}$.
   (a) Find the total number of zooplankton in the water column down to a depth of 4 m at both noon and midnight.
   (b) At which time is there greater numbers of zooplankton down to this depth?

2. Find the most general antiderivatives of the following functions

(a) $f(z) = z^5 - 2z^3 - 1$ (b) $h(y) = e^{2y} + \frac{1}{y^{2/3}}$ (c) $y(x) = 2\sin(3x) - \cos(x)$

3. Find the area under the curve $y = \frac{x}{1+x^2}$ from x=0 to x=2

4. New cases of cholera following an outbreak in a region are reported at a rate of $C(t) = 10t^2 e^{-t/5}$ where C(t) gives the rate of new cases per day on day t following the start of the outbreak. If there were 100 cases reported by day t = 0, what is the estimated total number of cases which have been reported by day t = 10?

5. Find the volume of the solid of revolution generated by revolving the curve $y = x^2 + x$ about the x-axis for $0 \le x \le 1$.

6. Evaluate the following integrals

(a) $\int_1^2 x\ln(x)\,dx$ (b) $\int_0^1 \frac{1}{\sqrt{2x+4}}\,dx$ (c) $\int_0^{\pi/2} x\sin(2x)\,dx$

7. The evening temperature during month t (where t = 0 corresponds to January 1 , t = 1 corresponds to February 1, etc.) in ºC in a city is approximated by the function

$T(t) = 12\sin(\frac{\pi}{6}(t-4)) + 14$

What is the average temperature in this city over 12 months?