Math 152 – Sample Exam 2 – Spring 2016 – Louis Gross

It would be best if you try to take this Sample Exam as if you were sitting in class, using only a calculator. For the actual in-class exam, there will be blank sheets of paper handed out for you to give the answers but it will be important for you to SHOW YOUR WORK even if you are certain your answer is correct. Note that this Sample Exam is designed to take one hour, approximately the same length as the actual exam will be. On the actual exam I will give you the formulas for derivatives below, but I expect that you will know all the other rules for finding derivatives in Chapters 18-20.

Table of Derivatives:

$D_x(mx+b) = m$

$D_x(Ce^{kx}) = Cke^{kx}$

$D_x(Ca^x) = \ln(a)Ca^x$

$D_x(\sin(Cx)) = C\cos(Cx)$

$D_x(\cos(Cx)) = -C\sin(Cx)$

$D_x(x^n) = nx^{n-1} \mathit{for}\ n \neq -1$

$D_x(\ln(x)) = \dfrac{1}{x}$

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1. Sketch the graphs of the following functions. State where all relative maxima and minima occur and give the location of any inflection points.

   (a) $y = \dfrac{x^4}{4} - \dfrac{3x^2}{2} + 1$ (b) $y = \dfrac{x^2-1}{x^2+1}$ (c) $f(x) = x + 2 - \dfrac{1}{x-2}$

2. The size of web a particular spider makes affects the daily food intake in some sites, with larger webs allowing more food to be captured. However, larger webs require more energy for the spider to build and maintain. For a circular web, assume that the amount of food captured per day is proportional to the radius of the web, but the maintenance and building cost is proportional to the web area. Suppose the daily food intake is 120 calories when the web has radius 10 cm and the cost is 30 calories per day when the web area is 100 cm^2. Find the web size that is optimal, which means that it maximizes the daily food intake minus the cost.

3. Find the largest and smallest values of each of the following functions on the given intervals and state the x-value at which these occur.

   (a) $f(x) = x + \dfrac{4}{x}\ on\ [1,3]$ (b) $f(x) = \dfrac{x}{x^2+1}\ on\ [-2,2]$

4. The change in a lizard's body temperature when the animal moves from sun to shade (this behavior is called shuttling) is found to be reasonably well described by Newton's Law of Cooling. Thus the rate of change of body temperature is proportional to the difference between body temperature and the temperature of the surroundings. Suppose a lizard's temperature is 30 °C when it is in the sun and drops to 20 °C after 10 minutes in the shade where the surrounding temperature is 15 °C. If T(t) gives the lizard's temperature at time t minutes after going from sun to shade, then
(a) Give an equation for T′(t) as well as the boundary condition.
(b) Find an equation for T(t) and use it to find T(20).

5. For fish such as salmon, the Ricker Model provides an estimate of the number of new "recruits", R which means the number of fish which will survive to become adults in the next generation, as a function of the number of fish, S, spawning in the previous generation. The model states that

$$R = aS\,e^{-bS}$$

where a and b are constants. For this model, find an equation for the level of spawners, S*, which would maximize the number of recruits.

6. State where each of the following functions is concave up, where it is concave down and where it has inflection points.

(a) $f(x) = x^2 e^{-x}$ (b) $g(x) = 6x^2 + 3\ln x \; for \; x > 0$

7. Find the derivative of each of the following functions.

(a) $f(x) = \ln(\sin^2(3x+2))$ (b) $g(x) = 4x^3 + 5x^2 - \dfrac{1}{\sqrt{3x}}$

8. Find the equation of the tangent line at x=2 for each of the following functions

(a) $h(x) = \dfrac{x^3}{4} + x$ (b) $g(x) = \cos(\dfrac{\pi x}{4})$

9. For the function $f(x) = x^3 e^{x^2}$, using the definition of the derivative, give a limit you could use to compute $f'(4)$.