

Math151 at the University of Tennessee, Knoxville - Chat for October 28, 2015 with the course instructor, Louis Gross.

I will be online starting at 8:00PM and will be happy to answer questions regarding any aspect of the course, assignments, etc. You can type in this document to ask questions.

When you ask a question, please do not use your name because this document will be saved and publicly posted after we close it. I will be on-line at least until 9:00PM but will stay on longer if there are still questions. Note that I do not know the identity of anyone posting questions - each participant shows up as "Anonymous" animal.

I am now online - Lou

For number two on the practice exam, I understand how you find the eigenvalues but I have no idea how to find the eigenvectors for each value.

OK I assume then that you know how to get the eigenvalues 3 and -1 and to find the eigenvector we use the definition that for a matrix A and an eigenvalue lambda that the

eigenvector v must satisfy  $Av = \lambda v$   $\begin{bmatrix} 2 & 6 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$  and then we just need to find the x and y that satisfy this - there is not a single answer - just the ratio of the two is unique. So from this we have

$2x + 6y = 3x$  which gives  $x = 6y$  so an eigenvector is any vector with the x value being 6 times

the y value so it looks for example like  $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$  this is an eigenvector for the eigenvalue 3 but it

would be just as correcto give it as  $\begin{bmatrix} 6/7 \\ 1/7 \end{bmatrix}$  or as

OK - now you do exactly the same thing to find the eigenvector for the eigenvalue -1 except you replace the 3 in the above equation with -1

Do you want me to do this one too?

Yes please

OK for the eigenvalue -1 the equation is  $\begin{bmatrix} 2 & 6 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} x \\ y \end{bmatrix}$  and from this looking at the first row we get  $2x + 6y = -x$  which gives  $x = -2y$  which means that any vector with the x value being -2 times the y value is an eigenvector for the eigenvalue -1 so we can choose for example  $y=1$

then the eigenvector is  $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

OK?

Can you work out #2 pls?

if you wouldn't mind

Could you tell us how to find the eigenvectors from the eigenvalues please?

I don't really understand the eigenvalues and eigenvectors, could you work that problem?

Not sure I know which problem you are referring to here.

On problem 1a, I keep getting a different answer from the key. Could you please explain the process for this one?

OK let me put this into the math code for this document  $4\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} - 3\begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}$   
so we are subtracting two matrices that are of the same size (they are both 2x2) so we are able to do this. Take the first matrix and multiply each entry by 4 and take the second matrix and multiply each entry by 3 to get  $\begin{bmatrix} 0 & 8 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -3 \\ 12 & 6 \end{bmatrix}$  then just subtract each entry in the second matrix from the corresponding entry in the first matrix to get the answer.

Got it! Thank You

Could you please explain how to get the eigenvalues 3 and -1 for Q2?

OK - I am going to assume that you know that the characteristic equation for the 2x2 matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  is a quadratic equation given by  $\lambda^2 - (a + d)\lambda + ad - bc = 0$

so for the matrix in problem 2  $a=2$   $b=6$   $c=.5$  and  $d=0$  so plugging these in we get  $\lambda^2 - 2\lambda - 3 = 0$  and this factors into  $(\lambda - 3)(\lambda + 1) = 0$  so there are two solutions 3 and -1  
Is this OK? Then to get the eigenvalues do what I showed above.

Ok, so im assuming we have to have that equation memorized.. is there a different equation for a 3x3 matrix that we should know as well for the exam?

Good questions - first yes you do need to know this formula for the characteristic equation for a 2x2 matrix - there are other ways to get the eigenvalues which we mention in the book but if you want the easiest way it is this formula. It only works for a 2x2 matrix though. We do an example of finding eigenvalues for a 3x3 matrix in the book but I don't expect you to memorize that method - it is why we have tools like Matlab. However I do expect that you know that for any size transfer matrix, including a 3x3, for which each of the columns of the matrix sum to one, the dominant eigenvalue is always one.

Alright, thank you.

Can you explain how to do 3D please?

Whenever you are asked about long-term growth rates you should think about the dominant eigenvalue because that is the long term growth rate. Similarly when you are asked about the long term population structure that is given by the eigenvector associated with the dominant eigenvalue. In this part we are asked to find the long term structure so we want the dominant eigenvalue first and then find the eigenvector for it. Do you understand how to get the dominant eigenvalue ( it is needed for part c of this)?

Yes I did part C but was just confused on part D

You can see this all worked out in the Chat document for October 21  
[Chat for October 21, 2015](#)  
is the link

Thank you!

Could you please explain the first part of question 3? I'm mainly confused on how to derive the matrix equation

OK - is it finding the projection matrix from the wording that you are having difficulty with or is it writing out a matrix equation?

If you could explain both I would really appreciate it

OK let's start with finding the projection matrix. First step is to determine how many classes there are. In this case we are told there are F and S stages so there are only two. This means the matrix that we use to project to the next year will be a 2x2 matrix. Then we need to use the wording to find the entries in the matrix. The first row of the matrix is the fecundities and we are told that only the S stage individuals produce offspring (90 seeds per year) so the entry in the first row second column is 90 and the entry in the first row first column is zero. If the F stage

individuals did produce offspring (seeds) then we'd have that number of seeds in the first row first column. OK so far?

Yes, so is the first row for this type of question always fecundities?

Yes, for the Leslie matrix projection model the first row gives the numbers of offspring so it is the fecundities.

Now the second row is the survivorships and we are told that only 10% of the F stage individuals survive to be S stage so the first column of the second row is .1 and we are also told that all S stage individuals die after one year in S stage (this is what happens for biennial plants - they live two years at most then die). The second column of the second row is zero because

all S stage individuals die and don't survive. So this is how we get the matrix  $\begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix}$  and then we just need to write the matrix equation that goes from time  $t = 0$  to time  $t = 1$

$$\begin{bmatrix} F_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix} \begin{bmatrix} F_0 \\ S_0 \end{bmatrix}$$

Is this OK? or are there more questions about it?

Thanks for the explanation...definitely have a better understanding now.

I'm actually having trouble understanding how to do the next part. I'm having trouble with the short term projections.

OK - for these you are just using an equation like the matrix one above and plugging in the

numbers for the  $\begin{bmatrix} F_0 \\ S_0 \end{bmatrix}$  so in the case of part b we are given this as  $\begin{bmatrix} 10 \\ 2 \end{bmatrix}$  (note that the problem says 2 F stage but that is a typo and it should have said 2 S stage - I assumed people knew that was a typo).

Then to get  $\begin{bmatrix} F_1 \\ S_1 \end{bmatrix}$  we just multiply  $\begin{bmatrix} F_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \end{bmatrix}$  and we get  $\begin{bmatrix} F_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 180 \\ 1 \end{bmatrix}$

then we do the same thing to get the answer for after 2 years

$$\begin{bmatrix} F_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 & 90 \\ .1 & 0 \end{bmatrix} \begin{bmatrix} 180 \\ 1 \end{bmatrix} \text{ which gives } \begin{bmatrix} F_2 \\ S_2 \end{bmatrix} = \begin{bmatrix} 90 \\ 18 \end{bmatrix}$$

OK?

yes I understand it now.

Can you please explain the matlab question? (Question 5)  
Basically, what types of matlab questions are fair game for this exam?

OK - we have gone over only really simple commands in matlab - so I expect that you know how to use Matlab to multiply two matrices (or a matrix times a vector which is shown in problem 5), and know what the commands are for finding eigenvectors and eigenvalues and what we did on the first exam (e.g. histograms, plotting for scatter plots). I don't expect that for this exam you will write any Matlab code, but as in the first exam and on the sample, I expect that you can explain what a few simple Matlab commands are doing. I am NOT going to give you a .m file and ask to explain each step, or anything with loops in it (that is , nothing with `for i= 1:10` etc. )

Does this help?

Yes, thanks

If there isn't anything else, I am going off-line

Btw thanks for having these sessions, they're very helpful. I wish more teachers would do this...

Might be difficult for English faculty to do this!!

That's true

Good night