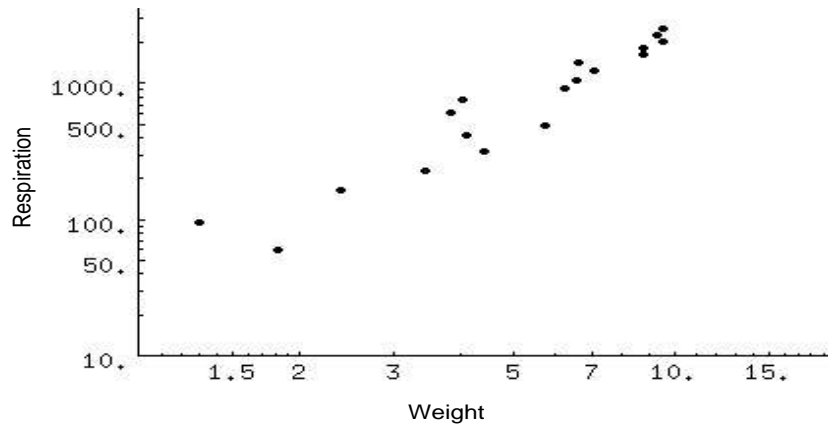


Math 151 - Sample Final Exam - Fall 2003 - Lou Gross

1. For the below figure showing the relationship between respiration (R) and weight (W) for a variety of species: (a) estimate the equation for a linear regression through the data, (b) express R as a function of W, and (c) if the weight of species A is twice the weight of species B, what is the ratio of the respiration rate of species A to that of species B?



2. For the below matrix, find all eigenvalues and their associated eigenvectors.

$$\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

3. Carry out the following matrix operations, if possible. If it is not defined, state so.

(a) $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 & 1 \\ 2 & 3 & 4 \\ 0 & 1 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 0 & 2 \\ 1 & 5 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 4 & 5 \end{pmatrix}$

4. Suppose a population consists of two groups, juveniles and adults, living for one year in each class. Juveniles do not reproduce, and on average one out of every four juveniles survives to become an adult. Adult females produce on average 8 offspring who are juveniles, before the adult dies.

(a) Give a matrix equation that allows you to predict the number of juvenile and adult females at any time, given that you know the initial numbers of these at time 0.

(b) Suppose there are 12 juveniles and one adult (all females) present at time 0. How many of each will be present two years later?

(c) If the population were to exist for a long time, what would its long-term growth rate be? What would be the long term ratio of juveniles to adults in the population?

5. A survey of cat owners indicates that 80% of cats will eat chicken flavored cat food, 40% will eat fish flavored food, and 30% will eat both of these. Find the probability a randomly selected cat will:

- (a) eat chicken or fish flavored food or both
- (b) eat chicken or fish flavored food but not both
- (c) not eat either chicken or fish flavored food.

6. An urn contains 5 Red, 2 Blue, and 1 Green ball.

(a) If a ball is drawn, then replaced and another ball is drawn, find the probability both balls drawn are Blue.

(b) If two balls are drawn without replacement, find the probability both balls drawn are Blue.

7. Twenty percent of a plant population grows in a nutrient rich habitat, and the rest of the population grows in a nutrient poor habitat. Plants growing in a nutrient rich habitat have a 20% chance of being infected by a fungus, while plants in nutrient poor sites have a 60% chance of being infected by the fungus.

(a) What fraction of the entire population is infected by the fungus?

(b) Given that you have found an infected plant, what is the probability it is on a nutrient rich site?

(c) What fraction of those plants not infected by the fungus are from nutrient rich sites?

8. Solve the following difference equations for x_n and find the value of x_4 :

(a) $x_{n+1} = 3x_n - 1$, $x_0 = 4$. (b) $x_{n+1} - x_n - 5 = 0$, $x_0 = 2$.

(b) $x_{n+1} - 2x_n - 3x_{n-1} + 16 = 0$, $x_0 = 2$, $x_1 = 14$.

(d) $x_{n+1} = 5x_n - 6x_{n-1} + 8 = 0$, $x_0 = 5$, $x_1 = 8$.

9. A fair, six-sided die is tossed 3 times. Find the probability of at least one 5 occurring on the 3 tosses.

10. During an algal bloom, the population of algae in a lake increases by 30% every day.

(a) Give an equation for the population density of algae in the lake during a bloom, where x_n = population density (algal cells / ml) and x_0 is the initial density of cells just as the bloom is starting.

(b) How many days will it take for the algal density to go up by a factor of three from its initial density?